# A CLASSIFICATION OF MODULES OVER COMPLETE DISCRETE VALUATION RINGS

BY CHANG MO BANG[1]

Communicated by Richard Pierce, October 17, 1969

1. **Introduction.** The purpose of this paper is to announce the completion of a classification (up to isomorphism) of all modules which are direct sums of countably generated modules over complete discrete valuation rings. The detailed proofs will appear elsewhere. Throughout this paper, let $R$ denote a fixed but arbitrary complete discrete valuation ring and $p$ a fixed but arbitrary prime element of $R$. For the sake of convenience, a cardinal is viewed as the first ordinal having that cardinality. Let $(c, R, k)$ be the class of all countably generated reduced $R$-modules of (torsion-free) rank $\leqq k$ and $D(c, R, k)$ that of all direct sums of members of $(c, R, k)$. Clearly

$$\begin{array}{ccccccc} (c, R, 0) & \subset & (c, R, 1) & \subset \cdots \subset & (c, R, \omega) \\ \cap & & \cap & & \cap \\ D(c, R, 0) & \subset & D(c, R, 1) & \subset \cdots \subset & D(c, R, \omega). \end{array}$$

Notice that a $p$-primary abelian group is a member of $(c, R, 0)$, particularly if $R$ is a ring of $p$-adic integers. A classification (of all members) of $(c, R, k)$ was done by Ulm (1933) when $k=0$ [**8**], by Kaplansky and Mackey (1951) when $k=1$ [**4**], by Rotman and Yen (1961) when $k<\omega$ [**7**], and that of $D(c, R, k)$ was done by Kolettis (1960) when $k=0$ [**5**]. First, we complete a classification of $(c, R, \omega)$ and then, utilizing this, we finish that of $D(c, R, \omega)$.

2. **Invariants.** We need two kinds of invariants, namely, the Ulm invariants and the basis types. Since the celebrated Ulm invariants are well known, a brief explanation of the basis types only is in order [2], [4], [7]. Let $R^k = \oplus\{R: i<k\}$ for each $k$. Define $f(R)$ to be the class of all sordinal (ordinal or $\infty$) valued functions on $R^k$ for all cardinals $k$, and $m(Q)$ that of all square row-finite matrices over $Q$, the quotient field of $R$. Suppose that $f, g \in f(R)$. Define $f \sim g$ to mean

*AMS Subject Classifications.* Primary 2030, 2027; Secondary 1340, 1350.

*Key Words and Phrases.* Countably generated module, complete discrete valuation ring, classification up to isomorphism, torsion-free rank, Ulm invariant, reduced module, $p$-adic integers, basis, basic free submodule, basis type, summandwise basis, height-preserving isomorphism, $p$-height.

[1] The author was supported in part by NSF Grant GP-8725 and GP-11804.