EXISTENCE OF GENERAL BARGAINING SETS FOR COOPERATIVE GAMES WITHOUT SIDE PAYMENTS

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1. **Introduction.** The concept of a bargaining set for cooperative games with side payments was introduced by Aumann and Maschler in [2]. In [5] and [9] a particular bargaining set was defined which has the property that for each partition of the players, there is a payoff which is in this set. In [10], Peleg shows that although this bargaining set generalizes naturally to games without side payments, the existence theorem is no longer true.

In this paper we prove an existence theorem for a general class of bargaining sets for games without side payments. The treatment is similar to that of Peleg in [11], and the proofs rely directly on Peleg's results in [9]. It is hoped that the work here will provide a way of satisfactorily generalizing the classical bargaining set to the class of games without side payments. Several attempts at this will be mentioned.

For a survey of work in the no side payment theory, see [1]; for work on a related solution concept, the core, see [4] and [13].

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2. **Definitions and main result.** Let the set of players be $N = \{1, \cdots, n\}$. For each $S \subset N$, let E^S be the Euclidean space of dimension $|S|$ whose coordinates are indexed by the players in S. If $u \in E^N$ then u^S will denote its projection onto E^S. If x and y are vectors we say $x \geqq y$ if $x \geqq y$ and $x \neq y$.

We use Ω_S and Ω_S^+ to denote respectively the nonnegative and the strictly positive orthant in E^S, i.e., $\Omega_S = \{x \in E^S \mid x \geqq 0\}$, and $\Omega_S^+ = \{x \in E^S \mid x > 0\}$.

For our purposes we will use the following definition of an n-person game with no side payments.

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