TWO REMARKS ON A. GLEASON'S FACTORIZATION THEOREM

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The theorem of A. Gleason [2, vii.23] asserts that every continuous map f from an open subset U of a product X of separable topological spaces into a Hausdorff space Y whose points are G_δ-sets has the form $g \circ \pi|_U$, where π is a countable projection of X and $g: \pi(U) \rightarrow Y$ is continuous. A natural question is to find what other "pleasant" subsets U of X have the above factorization property. The most plausible ones are compact subsets: for, if $U \subseteq X$ is compact and $f = g \circ \pi|_U$ with f continuous, then g must be continuous since $\pi|_U$ is a closed map (being continuous on a compact space).

The first part of this note rejects this conjecture by giving an example of a compact subset of a product of copies of the unit interval, without the factorization property. In the second part, it is proved that the factorization $f = g \circ \pi|_U$ always holds whenever f is uniformly continuous and the range metric. This result implies an open mapping theorem for continuous linear mappings on products of Fréchet spaces.

1. **The example.** Let Z be a compact Hausdorff space which is first countable but not metrizable. Such a space exists by [1, §2, Exercise 13]. Since Z is completely regular, Z is homeomorphic to a compact subset U of a product X of copies of $[0, 1]$. Let $f: U \rightarrow U$ be the identity. Assume that $f = g \circ \pi|_U$, with π a countable projection and $g: \pi(U) \rightarrow U$ continuous, and argue for a contradiction. Since countable products of separable metric spaces are separable metric, $\pi(U)$ is separable metric. Hence U is a continuous image of a separable metric space. But a cosmic metric space is metrizable whenever it is compact by [3, p. 994, (C) for cosmic spaces]. This contradicts the assumptions on Z.

2. **A factorization theorem.** The above example shows that the following result does not hold longer when Y is not metrizable.

THEOREM. *If Z is any subset of a product of arbitrary uniform spaces*

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