CONTINUOUS SELECTION OF REPRESENTING MEASURES[1]

BY H. S. BEAR

Communicated by Victor Klee, August 25, 1969

Let B be a linear subspace of $C_R(X)$, the continuous real functions on a compact space X, and let Γ be the Šilov boundary of B in X. We give here conditions which are sufficient for there to be an integral representation of the form

$$u(x) = \int_\Gamma u(\theta) g_x(\theta) d\mu(\theta), \tag{1}$$

where $x \to g_x$ is a continuous map from some subset Δ of X into $L_\infty(\mu)$. With the additional condition that Δ is separable, we obtain a kernel representation of the form

$$u(x) = \int_\Gamma u(\theta) Q(x, \theta) d\mu(\theta) \tag{2}$$

where Q is a continuous function of x and $x \to Q(x, \cdot)$ is continuous with respect to the $L_\infty(\mu)$ norm. If it is also the case that $B|\Gamma$ is dense in $L_1(\mu)$, then $Q(\cdot, \theta)$ is a limit (uniform convergence on compact subsets of Δ) of functions in B. These results also give integral representations like (1) and (2) for a complex function algebra, simply by considering the space B of real parts of the algebra. The details of this work will appear in [3].

We use the following notation throughout this paper:

X is a compact Hausdorff space, with topology $\mathfrak{T}$.

B is a linear subspace of $C_R(X)$, containing the constant functions, and separating the points of X.

Γ is the Šilov boundary of B in X.

$\overline{B}_\Delta$, for any set $\Delta \subset X$, is the closure of $B|\Delta$ in the topology of uniform convergence on compact subsets of Δ.

$B^+(\Delta, z) = \{u|\Delta : u \in B,\ u > 0,\ u(z) = 1\}$.

($\overline{B}_\Delta$ is an abstract version of "all harmonic functions on Δ," and $B^+(\Delta, z)$ is the set of normalized-at-z positive B-functions.)

AMS Subject Classifications. Primary 4625, 4655; Secondary 5200.

Key Words and Phrases. Linear subspace of $C_R(X)$, integral representation kernel representation, Gleason parts, part metric, function algebra.

[1] This research was supported in part by grant NSF GP 7681.