

A SURVEY OF INTEGRAL REPRESENTATION THEORY¹

BY IRVING REINER

TABLE OF CONTENTS

1. Introduction. Notation and definitions.....	159
2. General remarks. Jordan-Zassenhaus Theorem.....	162
3. Extensions.....	165
4. Higman ideal.....	168
5. Representations over local domains.....	171
6. Genus.....	173
7. Maximal orders.....	176
8. Further results on genera.....	182
9. Projective modules and relative projective modules.....	186
10. Grothendieck groups and Whitehead groups.....	189
11. Commutative orders and related results.....	196
12. Divisibility of modules.....	200
13. Hereditary orders and related results.....	201
14. Finiteness of the number of indecomposable representations.....	205
15. Representations of specific groups and orders.....	207
16. Representation rings.....	208
17. Group rings.....	210
18. Algebraic number theory.....	212
19. Krull-Schmidt and Cancellation Theorems.....	213

1. Introduction. Notation and definitions. First of all I wish to acknowledge with thanks the many helpful conversations I have had with Professors Olga Taussky, Peter Roquette and Hans Zassenhaus, when I first began studying the subject of integral representations.

Historically, the subject received its main impetus from two branches of algebra. One branch is algebraic number theory, especially that part concerned with ideal theory; and the other is matrix theory, mainly that portion dealing with matrix representations of associative algebras. Methods of homological algebra have played an increasingly important role in the subject in recent years.

An expanded version of an address delivered before the Chicago meeting of the Society by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, April 20, 1968, under the title *Recent progress in the theory of integral representations*; received by the editors September 17, 1969.

AMS Subject Classifications. Primary 1075, 1548, 2080, 1640; Secondary 1069, 1620.

Key Words and Phrases. Integral representations, orders, representation lattices, Grothendieck groups, noncommutative arithmetic, indecomposable representations, group representations, genus.

¹This work was partially supported by a research contract with the National Science Foundation.