INTEGRALS WITH NONDEGENERATE CRITICAL POINTS

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Palais [3] and Smale [6] have given conditions under which a function on a Hilbert manifold is a Morse function. Palais [4] and the author [7] have given classes of integrals on manifolds of sections of fiber bundles which satisfy the condition (C) of Palais and Smale and therefore satisfy a Ljusternik-Schnirelmann category theory [4], but for Morse theory the critical points must in addition be nondegenerate. We now outline a proof of a conjecture of Palais': that for almost all choices of Dirichlet boundary values, certain of these integral have only nondegenerate critical points on that boundary value manifold and therefore satisfy the full Morse theory. The model for these results and immediate corollary is the fact that the conjugate points for the geodesic problem on a Riemannian manifold are residual. For any two Riemannian manifolds M and N with $\partial M \neq \emptyset$, $\partial N = \emptyset$, and $2k > \dim M$, we construct a canonical function E_k on $H_k(M, N)$ which is a Morse function on $H_k(M, N)_\lambda$ for almost all λ.

We first prove an abstract theorem. Let $\pi: H \to G$ be a separable C^∞ fiber bundle over a Banach manifold G with a Banach manifold as fiber and identify $\pi^*T^*(G)$ with $(d\pi)^*T^*(G) \subseteq T^*(H)$. $H_g = \pi^{-1}(g)$ for $g \in G$.

THEOREM 1. *Let J be a C^2 function on H and consider the C^1 function $dJ: H \to T^*(H)$. If*

(1.1) *The Hessian of $J|H_g$ is a Fredholm operator at all critical points for all $g \in G$, and*

(1.2) $dJ \pitchfork \pi^*T^*(G)$

then the subset $\{g \in G \mid J|H_g$ has only nondegenerate critical points$\}$ is residual in G.

The proof of Theorem 1 follows directly from a theorem on transversality similar to the theorem given by Abraham in [1]. Since (1.2) is hard to verify directly, we state an equivalent condition (1.3).

(1.3) At critical points of $J|H_g$ for all $g \in G$ we have

$$d^2J_s: T_s(H) \to T_s(T^*(H)) \cong T_s^*(H) \oplus T_s(H) \xrightarrow{(di_g)^*} T_s^*(H_g)$$

where $i_g: H_g \subseteq H$, and we require $(di_g)^* \circ d^2J_s$ to be onto.

We use the definitions and terminology of Palais [5]. Let $p: E \to M$ be a smooth finite-dimensional fiber bundle over a compact n