FACTORING THE HILBERT CUBE

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In 1964, R. D. Anderson announced [1] that the product of a countably infinite collection of dendra is a Hilbert cube (a dendron being a nondegenerate, uniquely arcwise connected, Peano continuum). He and R. H. Bing conjectured in [4] that a countably infinite product of compact contractible complexes is a Hilbert cube. The purpose of this note is to announce the following theorems which affirm the above Anderson-Bing conjecture. The author wishes to thank R. D. Anderson for several helpful conversations on the subject dating as far back as 1966.

THEOREM 1. *A countably infinite product of nondegenerate spaces is a Hilbert cube if the product of each space with the Hilbert cube is a Hilbert cube.*

THEOREM 2. *If two finite simplicial complexes have the same simple homotopy type, then their products with the Hilbert cube are homeomorphic.*

For simply connected (finite) simplicial complexes, the concept of simple homotopy type coincides with that of homotopy type, since the algebraic invariant which distinguishes the two vanishes [10]. Thus, a contractible (finite) simplicial complex has the simple homotopy type of a point. Combining this with the above two theorems yields

COROLLARY 1. *A countably infinite product of nondegenerate, contractible, finite simplicial complexes is a Hilbert cube.*[1]

COROLLARY 2. *The product of a locally finite simplicial complex with the Hilbert cube is locally homeomorphic to the Hilbert cube.*

This is by Theorem 2 since each point of a locally finite simplicial complex has a neighborhood which is a collapsible complex.

The next is the converse of a theorem of D. W. Henderson, who proved in [7] that each open subset of a separable, infinite-dimensional Hilbert space is homeomorphic to the product of that space with a countable, locally finite simplicial complex. Corollary 3 ap-

[1] The author understands by letter that Andrej Szankowski, a student of Pełczyński, has also obtained this result.