ON THE RELATIONS BETWEEN TAUT, TIGHT AND HYPERBOLIC MANIFOLDS

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In [1], Professor Kobayashi defined hyperbolic and complete hyperbolic manifolds. In [2], Professor Wu defined tight and taut complex manifolds. The purpose of this paper is to show that these concepts are related in the following way:

complete hyperbolic $\Rightarrow$ taut

taut $\underset{\not\Leftarrow}{\Rightarrow}$ hyperbolic

hyperbolic $\Leftrightarrow$ tight (with respect to some metric)

It seems likely that taut implies complete hyperbolic, but I cannot prove that at the present time. Don Eisenman has obtained these results concurrently by a slightly different method.

We begin by recalling the definition of the Kobayashi pseudo-distance d_M associated to the complex manifold M. Let p and q be points in M. By a *chain* α from p to q, we mean a sequence $p=p_0, p_1, \cdots, p_k=q$ of points in M, points $a_1, \cdots, a_k$ in the unit disk $D=\{z\in C \mid |z|<1\}$ and holomorphic maps $f_1, \cdots, f_k$ of D into M with $f_i(0)=p_{i-1}$ and $f_i(a_i)=p_i$. The length $|\alpha|$ of α is defined by

$$|\alpha| = \sum_{i=1}^{k} d(0, a_i) = \sum_{i=1}^{k} \log \frac{1+|a_i|}{1-|a_i|}$$

where d is the Poincaré-Bergman distance on D. It is given by the metric $ds^2=dzd\bar{z}/(1-|z|^2)^2$. We set $d_M(p, q)=\inf_{\alpha\in A}|\alpha|$, where A is the set of all chains from p to q. It is easy to see that d_M is a pseudo-distance on M. If d_M is an actual distance, we say that M is *hyperbolic. M* is called *complete hyperbolic* if d_M is a complete metric, i.e., if all Cauchy sequences converge. Kobayashi (see [1, §8]) has shown that complete hyperbolic implies that all bounded subsets have compact closure.

It follows immediately from the definition of d_M and d_N, that if $f\colon M\to N$ is holomorphic and $p, q\in M$, then $d_N(f(p), f(q))\leqq d_M(p, q)$. The classical Schwarz-Pick lemma implies that the Kobayashi distance d_D on the unit disk D is the same as the Poincaré-Bergman distance d.