THE ASYMPTOTIC MANIFOLD OF A NONLINEAR SYSTEM OF DIFFERENTIAL EQUATIONS

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The linear system of differential equations

$$dy/dt = A(t)y \tag{1}$$

is known to determine the asymptotic behavior of the nonlinear system of differential equations

$$dx/dt = A(t)x + f(t, x) \tag{2}$$

provided $f(t, x)$ is "sufficiently small." Our results, which are another contribution to this area, are motivated by two recent studies. Brauer and Wong [1] have obtained quite general results on the asymptotic relationships between the solutions of (1) and (2). We significantly weaken the hypotheses of one of their results; see Theorem 1 below. Toroshelidze [4] considered the problem of perturbing the asymptotic manifold (see definitions below) of a nonlinear scalar equation. This concept is discussed formally and in a more general setting by using systems (1) and (2); some related problems are also considered.

The techniques used in the proofs are a combination of the well-known comparison principle and the Schauder-Tychonoff fixed point theorem. Fundamental in the application of the comparison principle is a scalar equation

$$dr/dt = \omega(t, r). \tag{3}$$

In the above equations it will be assumed that $A(t)$ is a real valued continuous $n \times n$ matrix defined on the interval $J = [0, \infty)$; $f(t, x)$ is a real continuous n-vector valued function defined on $J \times R^n$ where R^n is Euclidean n-space; $\omega(t, r)$ is nonnegative and continuous on $J \times J$ with $\omega(t, r)$ nondecreasing in r, $r > 0$, for each fixed $t \in J$. The fundamental matrix of (1) which is equal to the $n \times n$ identity matrix at $t = t_0$ will be designated by $Y(t)$. The symbol $|\cdot|$ will be used to denote any convenient vector norm.

The following theorem improves Theorem 3 of Brauer and Wong [1] by replacing a Lipschitz condition by a more general inequality.

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