ON ABSOLUTELY CONTINUOUS TRANSFORMATIONS

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1. **Introduction.** In this announcement, we examine absolutely continuous transformations T mapping the measure space (S, Σ, μ) onto the measure space (S', Σ', μ'). In order to obtain information about the change of measure induced by T, a weight function W' defined on $S' \times \mathfrak{D}$ is introduced, where $\mathfrak{D}$ is a certain subfamily of Σ. $W'(s', D)$ represents a weight assigned to the points in D which T maps into $s' \in S'$. We present structure theorems (Theorems 2 and 3) for weight functions which enable us to establish a transformation formula (Theorem 1) for integrals defined on the measure spaces. Theorem 1 includes all the existing transformation formulas for transformations which are absolutely continuous with respect to a real valued weight function. Moreover, the integrability condition necessary to ensure the existence of the formula is minimal, as we shall indicate in §3.

Rado and Reichelderfer [11] considered the case when the measure spaces are Euclidean n-space (both having the same dimension), with Lebesgue measurable sets and n-dimensional Lebesgue measure; T is a bounded continuous transformation defined on the bounded domain S. In particular, the weight function $\mu_e(s', T, D)$ generated by the topological index defined on indicator domains is used to define an essentially absolutely continuous transformation. Also the Banach indicatrix or crude multiplicity function $N(s', T, D)$ and the weight function $k(s', T, D)$ which counts the number of essential maximal model continua for (s', T, D) are treated in detail in [11]. In this classical setting, Craft [10] removed some conditions on the weight functions. Reichelderfer [13] developed a transformation theory for general measure spaces under certain standard hypotheses. Necessary and sufficient conditions were given in order that a transformation be absolutely continuous. In [14] it was shown that a large class of topological spaces satisfies these hypotheses; consequently, in this general topological setting the concepts of absolute continuity and generalized Jacobians can be effectively defined. Brooks [1], [3] developed the theory for integrals in Banach spaces and introduced a larger class of weight functions [2]; as a special case, signed weight functions may now be used when the spaces are oriented. Lebesgue decomposition theorems and measurability theorems for positive weight functions were considered by Chaney [6], [7], [8].