CHARACTERIZATIONS OF LINEAR GROUPS

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Introduction. In his address at the International Congress of Mathematicians at Amsterdam, 1954, Brauer [3] proposed a program of studying simple groups, particularly various linear groups, by giving the structure of the centralizers of elements of order 2. He proved among other things that if a simple group G contains an element j of order 2 such that the centralizer $C_G(j)$ is isomorphic to the centralizer of an element of order 2 in a group L which is either the simple group $L_2(q)$ or $L_3(q)$ (with some restriction on q), then G is in fact isomorphic to L, except for a few isolated exceptional cases. Thus he gave a characterization of the groups $L_2(q)$ or $L_3(q)$ in terms of the structure of the centralizer of an element of order 2. The work of Brauer was followed by a large number of investigations along the same direction. This paper is a further study in this direction of characterizing linear groups by the structure of the centralizers of involutions. Its purpose is to characterize the simple groups $L_n(q)$ when q is a power of 2. We shall also give a survey of known results.

1. **General remarks and a survey of the known results.** The problems we are interested in have an intimate bearing on the fundamental problem of classifying the finite simple groups. By a theorem of Feit-Thompson [10], a nonabelian simple group is of even order. Hence the centralizers of elements of order 2 are the subgroups of a simple group, which are always present and hopefully can be exploited.

Although almost no general proposition concerning the relationship between the structure of a simple group and the structures of the centralizers of elements of order 2 is known at present, there is some indication that the situation will be improved in the near future.

We shall consider more specific situations. One formulation of the problem will be as follows: Let G be a simple group such that the centralizers of elements of order 2 satisfy a group theoretical property

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The original content of the address has been published in part [32]. Theorems proved in this paper are generalizations of the results of [32, IV] in higher dimensional case. The papers [32, V, VI] are yet unpublished, but the proofs are similar to that of [36].

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