

NONCLASSICAL SIMPLE LIE ALGEBRAS¹

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Introduction. Let Φ be an algebraically closed field of characteristic $p > 3$. In addition to the finite dimensional classical simple Lie algebras [12] over Φ a number of families of finite dimensional nonclassical simple Lie algebras over Φ have been discovered [1]–[3], [5]–[9], [13]. Until recently no general connection has been known between these algebras and any family of Lie algebras over fields of characteristic 0.

Recently Kostrikin and Shafarevitch [11] have given a unified construction of all known finite dimensional nonclassical simple restricted Lie algebras over Φ . These algebras are obtained as the analogues in prime characteristic of the simple infinite Lie algebras of Cartan type over \mathbf{C} .

We give here a generalization of the Kostrikin-Shafarevitch construction which gives all known finite dimensional nonclassical simple (not necessarily restricted) Lie algebras over Φ , as well as some which are new.²

I. Definition of Lie algebras of Cartan type. The infinite Lie algebras of Cartan type are certain Lie algebras over \mathbf{C} which arise in the study of pseudogroups [10], [15]. They are characterized by the following conditions:

- (1) L has a decreasing filtration $L = L_{-1} \supset L_0 \supset L_1 \supset \dots$.
- (2) $\bigcap L_i = (0)$.
- (3) $[L_i, L_j] \subseteq L_{i+j}$ for $-1 \leq i, j$ (where $L_{-2} = L$).
- (4) If $x \in L_i$ and $x \notin L_{i+1}$ for some $i \geq 0$ then there exists $y \in L$ such that $[xy] \notin L_i$.
- (5) $\dim L_{-1}/L_0 < \infty$.
- (6) $\dim L = \infty$.

¹ These results are contained in the author's doctoral dissertation written under the guidance of Professor G. B. Seligman at Yale University. The author was a National Science Foundation Graduate Fellow at Yale.

² *Added in proof.* In a recent paper (*Graded Lie algebras of finite characteristic*, *Izv. Akad. Nauk SSSR Ser. Mat.* **30** (1969), 251–322) Kostrikin and Shafarevitch have also studied the nonrestricted case and have obtained results which substantially overlap those of this paper.