GROUPS OF DIFFEOMORPHISMS AND THE SOLUTION OF THE CLASSICAL EULER EQUATIONS FOR A PERFECT FLUID

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1. **Introduction.** We announce several results on the structure of the group of diffeomorphisms $\mathfrak{D}$ of a compact n-manifold M, possibly with boundary. The group $\mathfrak{D}$ has the structure of a differentiable manifold modelled on a Fréchet space and with this structure, the group operations are smooth. See Leslie [5] and Omori [8], for the proof in case M has no boundary. Following Omori, we call $\mathfrak{D}$ an ILH Lie group.

We shall show that several infinite dimensional subgroups of $\mathfrak{D}$ are actually (ILH) submanifolds and hence also have the structure of ILH Lie groups. Also, we construct certain (weak) Riemannian structures on $\mathfrak{D}$ (and on certain subgroups) and find the geodesic flows associated to them.

More specifically, if μ is a smooth volume on M and $\mathfrak{D}_\mu$ consists of all diffeomorphisms leaving μ invariant then $\mathfrak{D}_\mu$ is a closed submanifold of $\mathfrak{D}$. This group $\mathfrak{D}_\mu$ hence has an ILH Lie group structure. This group is of fundamental importance in the study of hydrodynamics of perfect fluids in M, as it is the appropriate configuration space. Arnold [1] has shown that the solution of the classical Euler equations for the fluid corresponds to geodesics on $\mathfrak{D}_\mu$ relative to a given invariant metric on $\mathfrak{D}_\mu$ induced by a Riemannian structure on M. (See Marsden and Abraham [6] for a precise proof of Arnold's theorem.) We prove the existence of such a smooth geodesic flow and hence that the Euler equations have unique smooth solutions, existing for short time, and varying smoothly with respect to the initial conditions. We expect that the flow is complete and thus the solutions can be uniquely extended for all time. Weaker versions of the above result were proven by Kato [4] and others for two and three dimensional regions in Euclidean space, although Kato [4] shows that in the two dimensional case, solutions do exist for all time.

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2. **Technicalities.** In order to prove our results we must enlarge the group $\mathfrak{D}$ slightly so that it becomes a Hilbert manifold. We define $\mathfrak{D}^s$ (called the set of H^s diffeomorphisms) to be the set of all maps η: