KLEIN SURFACES AND REAL ALGEBRAIC FUNCTION FIELDS[1]

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Communicated by Serge Lang, February 21, 1969

Introduction. The correspondence between compact Riemann surfaces and function fields in one variable over C is well known and has been widely exploited, both in analysis and in algebraic geometry. A similar correspondence exists between function fields in one variable over R and compact "Klein surfaces," which are more general than Riemann surfaces in two respects: they need not be orientable and they may have boundary (see the precise definition below). But while this latter correspondence has apparently been in the folklore for some time, it seems to have been neither systematically expounded nor applied.

It was first noted by Felix Klein in 1882 [3] that nonorientable surfaces could carry an analytic structure. The analytic theory of Klein surfaces was treated in depth by Schiffer and Spencer in 1954 [4]. Our approach is quite different, since we endeavor to work within the category of Klein surfaces, rather than to lift problems to the category of Riemann surfaces by doubling. This approach allows us to prove certain results (such as Theorems 5 and 6 below) without resort to the technique of descent.

In this announcement we state the foundational definitions and theorems, and then give several applications, some of which appear to be new. A detailed monograph is in preparation.

1. **Klein surfaces.** Let D be an open set in C and let $f: D \to C$ be a C^2 function as a function of R^2 into R^2. f is of course analytic if $df/d\bar{z} = 0$ and antianalytic if $df/dz = 0$. We call f *dianalytic* if on each component of D it is analytic or antianalytic. If D is an arbitrary subset of C and $f: D \to C$, we say that f is *dianalytic* if there is a dianalytic extension of f to some open neighborhood of D.

Let X be a connected two-manifold with boundary ∂X (which may be empty). A *dianalytic coordinate covering* (d.c.c.) of X is a family $\mathfrak{U} = (U_j, z_j)_{j \in J}$, where $(U_j)_{j \in J}$ is an open cover of X, and z_j is a homeomorphism of U_j onto an open set in the closed upper half plane C^+, such that the transition functions $z_j z_k^{-1} | z_k(U_k)$ are all dianalytic. A *dianalytic* structure on X is a maximal d.c.c. $\mathfrak{X}$, and a *Klein surface*

[1] This research was supported, in part, by NSF grant GP 7548 and NSF grant GP 7436.