A WEIERSTRASS THEOREM FOR NORMED LINEAR SPACES

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1. **Introduction.** Let X and Y be real normed linear spaces and let K be a compact subset of X. Let $C(K, Y)$ denote the family of continuous functions from K to Y and let $\tilde{C}(K, Y)$ denote the family of continuous functions from X to Y restricted to K. Both $C(K, Y)$ and $\tilde{C}(K, Y)$ carry the uniform norm topology given by

$$\|f - g\|_u = \sup_{x \in K} \|f(x) - g(x)\|.$$

In the event $X = Y = R$, where R denotes the real numbers, the classical Weierstrass Theorem states that the family of real-valued polynomials on X is dense in $C(K, Y)$ where $K = [a, b]$. If $X = R^n$ and $Y = R$, an application of the Stone-Weierstrass Theorem states that the family of polynomials in n real variables, $n = 1, 2, \cdots$, is dense in $C(K, Y)$. Various generalizations of the Stone-Weierstrass Theorem have been given, notably by Hewitt, Hewitt and Zuckerman, de Branges, Kelley, and Buck. Buck's paper is notable in that he gives some attention to polynomial approximation to functions whose range is finite dimensional. The extension of the Weierstrass theory to a real separable Hilbert space setting was given by the author in [4] using the polynomial operators treated by Rall [7]. In that paper [4] it was suggested how the theory could be extended to a complex separable Hilbert space.

The purpose of the present note is to consider a Weierstrass Theorem for normed linear spaces. We will restrict ourselves to a brief outline of the theory of polynomial operators and a statement of some of the main results whose proofs together with other details and applications will appear elsewhere.

2. **Basic definitions.** Let X and Y be real or complex linear spaces. For each positive integer n, let X^n denote the direct product of X with itself n times. An *n-linear operator* L_n from X^n to Y is a function which is linear and homogeneous in each of its arguments separately. That is,

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