A NOTE ON FUNCTORS Ext OVER THE RING Z[1]

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Let A and B be modules over the ring Z of all integers. In this paper, we shall define a new homomorphism

$$\Gamma\colon B \otimes_Z \mathrm{Hom}_Z(A, Q/Z) \to \mathrm{Ext}^1_Z(A, B)$$

by $\Gamma(b\otimes h) = bE_0h$, for each $b\otimes h \in B\otimes_Z \mathrm{Hom}_Z(A, Q/Z)$ and check the properties of Γ, where $E_0\colon 0\to Z\to Q\to Q/Z\to 0$ is the familiar exact sequence and Q is the field of all rational numbers.

For convenience, in sequel we shall use $\otimes$, Hom and Ext for $\otimes_Z$, Hom_Z and Ext^1_Z, respectively, and A, B as Z-modules.

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The detailed definition of Γ is described by the diagram with each commutative square

$$\begin{array}{lccccccccl}
bE_0h\colon & 0 & \to & B & \longrightarrow & D_2 & \longrightarrow & A & \longrightarrow & 0\text{(exact)} \\
 & & & b\uparrow & \text{push-out} & \uparrow & & \| & & \\
 & 0 & \to & Z & \longrightarrow & D_1 & \longrightarrow & A & \longrightarrow & 0\text{(exact)} \\
 & & & \| & & \downarrow & \text{pull-back} & \downarrow h & & \\
E_0\colon & 0 & \to & Z & \longrightarrow & Q & \longrightarrow & Q/Z & \longrightarrow & 0\text{(exact)}
\end{array}$$

for each $b\otimes h \in B\otimes \mathrm{Hom}(A, Q/Z)$, where $b\in B$ is a homomorphism from Z to B such that $b(1) = b$.

By the standard methods as in [3] we know that for b_i $(i=0, 1, 2)$ in B and h_i $(i=0, 1, 2)$ in $\mathrm{Hom}(A, Q/Z)$ $(b_1+b_2)E_0h_0 = b_1E_0h_0 + b_2E_0h_0$, $b_0E_0(h_1+h_2) = b_0E_0h_1 + b_0E_0h_2$. Furthermore, for each $f\colon A_2\to A_1$ and $g\colon B_1\to B_2$, where A_i $(i=1, 2)$ and B_i $(i=1, 2)$ are Z-modules, we get the Z-homomorphisms

$$f^*_H\colon \mathrm{Hom}(A_1, Q/Z)\to \mathrm{Hom}(A_2, Q/Z), \qquad f^*_E\colon \mathrm{Ext}(A_1, B)\to \mathrm{Ext}(A_2, B)$$

$$g^*_E\colon \mathrm{Ext}(A, B_1)\to \mathrm{Ext}(A, B_2)$$

and in this case we also know that for each $b\otimes h_1 \in B\otimes \mathrm{Hom}(A_1, Q/Z)$ and $b_1\otimes h \in B_1\otimes \mathrm{Hom}(A, Q/Z)$

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