THE SPECTRUM OF NONCOMPACT G/Γ AND THE COHOMOLOGY OF ARITHMETIC GROUPS

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Introduction. The purpose of this note is to announce a theorem in the representation theory of semisimple groups (Theorem 1.2, below). This theorem implies that certain spaces of square summable harmonic forms on noncompact locally symmetric spaces, associated with $\boldsymbol{Q}$-rank one arithmetic groups, are finite dimensional. Assertion (1.3) then gives information about the boundary behavior at ∞ of such forms. Combining (1.3) with the computations in [4] and Raghunathan's square summability criterion in [6], we obtain upper bounds for some betti numbers of locally symmetric spaces associated with $\boldsymbol{Q}$-rank one arithmetic groups (these spaces are noncompact, but have the homotopy type of a finite simplicial complex (see [7])). In some cases we obtain vanishing theorems for the first and second betti numbers. For the first betti number, such a vanishing theorem was obtained in greater generality by D. A. Kazdan (see [3]) by a different method. We remark that Raghunathan's square summability criterion has been generalized to arbitrary $\boldsymbol{Q}$-rank in [1]. Therefore an extension of Theorem 1.2 to arbitrary $\boldsymbol{Q}$-rank would yield a corresponding extension of our present results on cohomology. A detailed proof of Theorem 1.2 and a full discussion of the application of this theorem to the cohomology of arithmetic groups will appear elsewhere. I wish to express my thanks to S. T. Kuroda and M. S. Raghunathan for stimulating discussions.

We now introduce some notation. Let $\boldsymbol{Q}$, $\boldsymbol{R}$, and $\boldsymbol{C}$ denote the fields of rational, real, and complex numbers, respectively, and let $\boldsymbol{Z}$ denote the ring of rational integers. Let G denote a connected, linear, semisimple, algebraic group which is defined and simple over $\boldsymbol{Q}$. For a subring $A \subset \boldsymbol{C}$, let G_A denote the A-rational points of G. However, when $A = \boldsymbol{R}$, we let $G = G_{\boldsymbol{R}}$. We let $\mathfrak{g}$ denote the Lie algebra of G, $\mathfrak{g}_C$ the complexification of $\mathfrak{g}$, and $\mathfrak{G}$ the universal enveloping algebra of $\mathfrak{g}_C$. We make the convention that $\mathfrak{g}$ is the space of right invariant vector fields on G. Hence $\mathfrak{G}$ is the space of right invariant differential operators on G. We denote the center of $\mathfrak{G}$ by $\mathfrak{Z}$. As is well known, $\mathfrak{Z}$ may be identified with the space of (adjoint-)invariant polynomials

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