A LOCAL SPECTRAL THEORY FOR OPERATORS. II

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I. **Introduction.** Let T be an operator on a Banach space B. Let $\sigma(T)$, the spectrum of T, lie on a line, a circle, or, more generally, a smooth curve. If the resolvent $R_z(T)=(T-zI)^{-1}$ satisfies a growth condition with respect to $\sigma(T)$, it is possible, in many cases, to develop an invariant subspace decomposition for T. We mention explicitly the work of Bartle [1], Godement [4], Leaf [6], Lorch [7], Schwartz [13], Wermer [19], and Wolf [20]. Since, in the references cited, none of the subspaces is necessarily complemented by an invariant subspace, one can not expect this invariant subspace decomposition to generate a countably additive resolution of the identity. Such a spectral resolution is precisely the achievement of the Dunford theory [3], but there it was necessary to assume a second condition in order to obtain it. This condition (Dunford Boundedness) is not easy to verify in practice.

In this note, we will study several situations in Hilbert space, where a strong growth condition on the resolvent is sufficient to guarantee a countably additive resolution of the identity, i.e., the operator turns out to be similar to a normal operator. The results in §3 generalize, and are dependent on, some recent work of Gokhberg and Krein. We will only sketch proofs. Complete details will appear in [16] and elsewhere.

From now on, the underlying space is always a Hilbert space. All operators are bounded. By a smooth Jordan curve, we mean a Jordan curve of class C^2 (in the complex plane).

II. In this section we study conditions on the resolvent which insure normality.

LEMMA 1. *Let* $\|(T-\lambda)^{-1}\|\leqq 1/d$ *where* $0<d<|\lambda|$. *Then*

$$\left\|\left(T^{-1}-\frac{\bar{\lambda}}{|\lambda|^2-d^2}\right)^{-1}\right\|\leqq\frac{|\lambda|^2-d^2}{d}.$$

THEOREM 1. *Let U be an open set and let $\sigma(T)\cap U$ lie in the smooth Jordan curve C. Let $\|R_\lambda(T)\|\leqq 1/\text{dist}\,[\lambda, C]$ for $\lambda\in U$. Then $T=T_1\oplus T_2$ where T_1 is normal, $\sigma(T_1)=$closure $[\sigma(T)\cap U]$ and $\sigma(T_2)\subset\sigma(T)\cap U'$.*

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