ON THE TRIANGULATION OF MANIFOLDS AND THE HAUPTVERMUTUNG

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1. The first author's solution of the stable homeomorphism conjecture [5] leads naturally to a new method for deciding whether or not every topological manifold of high dimension supports a piecewise linear manifold structure (triangulation problem) that is essentially unique (Hauptvermutung) cf. Sullivan [14]. At this time a single obstacle remains[3]—namely to decide whether the homotopy group $\pi_3(\mathrm{TOP/PL})$ is 0 or Z_2. The positive results we obtain in spite of this obstacle are, in brief, these four: any (metrizable) topological manifold M of dimension $\geqq 6$ is triangulable, i.e. homeomorphic to a piecewise linear ($=$PL) manifold, provided $H^4(M; Z_2)=0$; a homeomorphism $h: M_1 \to M_2$ of PL manifolds of dimension $\geqq 6$ is isotopic to a PL homeomorphism provided $H^3(M; Z_2)=0$; any compact topological manifold has the homotopy type of a finite complex (with no proviso); any (topological) homeomorphism of compact PL manifolds is a simple homotopy equivalence (again with no proviso).

R. Lashof and M. Rothenberg have proved some of the results of this paper, [9] and [10]. Our work is independent of [10]; on the other hand, Lashof's paper [9] was helpful to us in that it showed the relevance of Lees' immersion theorem [11] to our work and reinforced our suspicions that the *Classification theorem* below was correct.

We have divided our main result into a *Classification theorem* and a *Structure theorem.*

(I) CLASSIFICATION THEOREM. *Let M^m be any topological manifold of dimension $m \geqq 6$ (or $\geqq 5$ if the boundary ∂M is empty). There is a natural one-to-one correspondence between isotopy classes of PL structures on M and equivalence classes of stable reductions of the tangent microbundle $\tau(M)$ of M to PL microbundle.*

(There are good relative versions of this classification. See [7] and proofs in §2.)

Explanations. Two PL structures Σ and Σ' on M, each defined by a PL compatible atlas of charts, are said to be *isotopic* if there exists a

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[3] See note added in proof at end of article.