Linear and quasilinear elliptic equations, by O. A. Ladyženskaja and N. N. Ural'ceva. Translated by Scripta Technica, Math. in Science and Engineering, vol. 46, Academic Press, New York, 1968. xviii +495 pp. $24.00.

The present book could be aptly subtitled: "How to choose the right test function" for it is an outstanding treatise devoted mainly to estimates achieved through intricate choices of test functions in the weak or integral forms of the various elliptic equations under consideration. The results and methods are largely due to the authors, although many of their techniques have roots in the classical works of Bernstein and the more recent work of De Giorgi. The book is concerned with the following facets of the theory of second order, linear and quasilinear elliptic equations:

(i) the solvability of the Dirichlet problem in various both classical and generalized formulations;

(ii) interior estimates of solutions and their derivatives;

(iii) global estimates of solutions and their derivatives in bounded domains;

(iv) interior regularity of solutions;

(v) global regularity of solutions in bounded domains.

Naturally, these themes are interwoven. The derivation of estimates can generally be adapted so as to provide regularity results. The solvability of the Dirichlet problem is reduced to a question of a priori global estimates through the application of topological fixed point theorems in the appropriate Banach spaces.

The types of second order equations examined are distinguished according to whether or not they are (a) linear or (b) of divergence structure. Accordingly four different types are treated.

$$(1)\qquad Lu = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left[a_{ij}(x)u_{x_j} + a_i(x)u\right] + \sum_{i=1}^{n} b_i(x)u_{x_i} + a(x)u = f - \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i}.$$

$$(2)\qquad Lu = \sum_{i,j=1}^{n} a_{ij}(x)u_{x_ix_j} + \sum_{i=0}^{n} a_i(x)u_{x_i} + a(x)u = f(x).$$

$$(3)\qquad \sum_{i=1}^{n} \frac{d}{dx_i}\left[a_i(x, u, u_x)\right] + a(x, u, u_x) = 0.$$

$$(4)\qquad \sum_{i,j=1}^{n} a_{ij}(x, u, u_x)u_{x_ix_j} + a(x, u, u_x) = 0.$$