ON A GENERALIZATION OF THE GAMMA FUNCTION AND ITS APPLICATION TO CERTAIN DIRICHLET SERIES

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In this paper, we will introduce certain Dirichlet series which generalize Epstein's Zeta function. We prove that these Dirichlet series possess analytic continuation as meromorphic functions. In obtaining the analytic continuation, we introduce certain integrals which generalize the Gamma function and establish the analytic property of these integrals.

We consider Dirichlet series of the type

$$\zeta(s) = \sum_{\gamma \in Z^n - N_F} h(\gamma) F(\gamma)^{-s} \quad [2]$$

where $F(X) \in R[X]$ is a polynomial of degree $d\ (>0)$, $X = (X_1, \cdots, X_n)$, such that $F_d(X)$, the highest homogeneous part of $F(X)$, vanishes only at the origin, $N_F = \{x \in R^n \mid F(x) = 0\}$; and $h(x)$ is in $SP(R^n)$ which is the smallest subring of $C^\infty(R^n)$ containing $R[X]$ and $S(R^n)$, the Schwartz space. It is easy to see that $SP(R^n)$ is the ring containing all functions $h(x)$ of the form $h(x) = f(x) + G(x)$ where $f(x) \in S(R^n)$ and $G(X) \in R[X]$.

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1. We write

$$F(X) = F_d(X) + \cdots + F_0(X)$$

where $F_q(X)$, $q = 0, 1, \cdots, d$, are homogeneous parts of $F(X)$ of degree q. For $x = (x_1, \cdots, x_n) \in R^n$, we put $|x| = (x_1^2 + \cdots + x_n^2)^{1/2}$. Let $S^{n-1} = \{x \in R^n \mid |x| = 1\}$.

We shall consider the following three conditions:

(A) N_F is compact,

(B) N_{F_d} is compact,

(C) $F(X)$ is homogeneous and N_F is compact.

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[1] The results announced here are contained in the author's doctoral thesis written under the guidance of Professor Takashi Ono at the University of Pennsylvania.

[2] For subsets A, B in a set, $A - B$ denotes the set of points in A but not in B.