EIGENFUNCTION EXPANSIONS AND SIMILARITY FOR CERTAIN NONSELFADJOINT OPERATORS[1]

BY CHARLES GOLDSTEIN

Communicated by M. H. Protter, January 7, 1969

1. Let A_0 denote a selfadjoint operator acting in a Hilbert space H with spectral resolution $E_{0\lambda}$. We assume that for a fixed open interval S, the operator A_0E_{0S} has an absolutely continuous spectrum with uniform spectral multiplicity. Let $A=A_0+V$ be a closed operator acting in H such that $D(V)\supseteq D(A_0)=D(A)=D(A^*)\subseteq D(V^*)$ and $A^*=A_0+V^*$. Suppose that there exists an ϵ_0 such that for $0<\epsilon<\epsilon_0$ and λ in S, we have $\lambda\pm i\epsilon\in\rho(A)$, $\rho(A)$ denoting the resolvent set of A. We wish to construct two complete, orthonormal sets of eigenfunctions for the continuous part, A^c, of the operator A given a complete, orthonormal set of eigenfunctions for A_0^c.

We first define the notion eigenfunction expansion. Suppose that B is a locally convex linear topological space, dense in H, such that the injection mapping from B into H is continuous. Let B^* denote the dual space of B. $(\ ,\ )_H$ and $\|\ \|_H$ will represent the inner product and norm in H and $\langle u, w\rangle$ will denote the sesquilinear pairing of B^* and B ($w\in B^*$, $u\in B$).

DEFINITION. Let $(\Omega, d\omega)$ denote a σ-finite measure space. Suppose that there is a mapping $w^0(\lambda,\omega)$ from $S\times\Omega$ into B^* such that the transformation T^0 given by $T^0u=\mathfrak{u}^0(\lambda,\omega)=\langle u, w^0(\lambda,\omega)\rangle$ for each u in B may be extended to an isometry from $H_S^0=E_{0S}H$ onto $\mathcal{H}_S=L_2(S\times\Omega)$. Also, suppose that $(A_0\mathfrak{u})^0(\lambda,\omega)=\lambda\mathfrak{u}^0(\lambda,\omega)$ for each u in $D(A_0)$ and a.a. points (λ,ω) in $S\times\Omega$. We shall refer to the mappings $w^0(\lambda,\omega)$ as eigenfunctions and to T^0 as a spectral mapping of A_0E_{0S}. We say that the set $\{w^0(\lambda,\omega)\}$ gives an eigenfunction expansion for the operator A_0E_{0S}.

In §2 we give sufficient conditions for the operator AE_S to possess two eigenfunction expansions, where E_S denotes a projection operator to be constructed presently. Employing these eigenfunctions, we establish the similarity of the operators A_0E_{0S} and AE_S as well as a time-dependent scattering theory. In §3 we apply these results to differential operators as well as to "gentle" perturbations. In this note we confine ourselves to the statement of results. All of the results will be proven in a subsequent publication.

[1] Work performed under the auspices of the U. S. Atomic Energy Commission.