LEES' IMMERSION THEOREM AND THE TRIANGULATION OF MANIFOLDS

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In [4] Lees proves the following immersion theorem for topological manifolds: Let M, M', Q be topological manifolds, M a compact locally flat submanifold of the open manifold M', with $\dim M' = \dim Q = q$, and $\partial Q = \varnothing$. Write $\mathrm{Im}_{M'}(M, Q)$ for the s.s. complex of M' immersions of M in Q; and write $R(TM'/M, TQ)$ for the s.s. complex of representative germs of TM'/M in TQ. A representative germ is a bundle map of the tangent bundle TM' of M', restricted to a neighborhood of M, into the tangent bundle TQ of Q. Two germs are identified if they agree over a common neighborhood of M.

THEOREM (LEES). *If M has a handle decomposition with all handles of index $<Q$; the differential d: $\mathrm{Im}_{M'}(M, Q) \to R(TM'/M, TQ)$ is a homotopy equivalence.*

We show here how to simplify some of the hypotheses of this theorem and give applications to the problem of triangulating topological manifolds.

THEOREM A. *In the following two cases, the assumption that M has a handle decomposition may be dropped in Lees' Immersion Theorem.*

(1) $\dim M < \dim Q$.

(2) $\dim M = \dim Q \geqq 5$, *and Q is a piecewise linear (PL) manifold.*

Of course, if M is a PL-manifold, M has a handle decomposition, and hence Lees' theorem applies.

THEOREM B. *In the following cases, $R(TM'/M, TQ)$ may be taken to be the s.s. complex of ordinary bundle maps of TM', restricted to M, into TQ.*

(1) $\dim M = \dim Q$.

(2) $\dim M < \dim Q$, *M a closed submanifold of M' and M the homotopy type of a locally finite simplicial complex.*

We will say that an R^k-bundle $\mathcal{E}$ over a space dominated by a locally finite simplicial complex K admits a PL-bundle structure, if the pullback of $\mathcal{E}$ over K is the underlying topological bundle of a PL-R^k-

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