CONSTRUCTION OF WEAK SOLUTIONS OF THE NAVIER-STOKES EQUATION IN A NONCYLINDRICAL DOMAIN[1]

BY HIROSHI FUJITA AND NIKO SAUER

Communicated by Jürgen K. Moser, October 4, 1968

1. **Introduction and the result.** Consideration of the motion of a viscous fluid in a vessel with moving walls or in a vessel containing rigid bodies moving through the fluid leads us to the initial value problem for the Navier-Stokes equation in a noncylindrical domain in (t, x)-space. This problem will be denoted by (Pr. NC). Let $\Omega(t) \subset R^m$ ($m=2$ or 3) be the domain filled by the fluid at time t and let $\Gamma(t)$ be the boundary of $\Omega(t)$. We shall be concerned with the flow for the time interval $[0, T]$ $(T>0)$. We put

$$\hat{\Omega} = \bigcup_{t\in[0,T]} \Omega(t) \quad \text{and} \quad \hat{\Gamma} = \bigcup_{t\in[0,T]} \Gamma(t).$$

(Pr. NC) in its classical form is to find out the velocity field $u=u(t, x)$ and the pressure $p=p(t, x)$ which satisfy the following.

$$\begin{aligned} u_t &= \Delta u - \nabla p - (u\cdot\nabla)u + f(t,x) && \text{in } \hat{\Omega}, \\ \nabla\cdot u &\equiv \operatorname{div} u = 0 && \text{in } \hat{\Omega}, \\ u &= \beta(t,x) && \text{on } \hat{\Gamma}, \\ u &= a(x) && \text{in } \Omega(0). \end{aligned}$$

f, β and a are given (vector) functions. Here and hereafter the differential operators Δ and ∇ mean those for x variables only. The special case of (Pr. NC) with $\Omega(t)$ independent of t will be denoted by (Pr. C). The objective of the present note is to extend E. Hopf's existence theorem (cf. [1]) for weak solutions from (Pr. C) to (Pr. NC). Intending to emphasize rather the straightforwardness of the method than the generality of the result, we here make the simplifying assumptions (A1)–(A3). These assumptions will be released completely or weakened considerably in a forthcoming paper where we shall give full details of our study.

[1] This research was partly supported by the National Science Foundation, Grant NSF-GP-8114 while the authors were visiting members at the Courant Institute of Mathematical Sciences. The first author was subsequently supported as a visiting professor in the Mathematics Research Center, United States Army, Madison, Wisconsin, under Contract No. DA-31-124-ARO-D-462.