CANONICAL THEORY OF THE NONPARAMETRIC LAGRANGIAN MULTIPLE INTEGRAL PROBLEMS WITH VARIABLE BOUNDARIES[1]

BY HANS RUD. WEBER

Communicated by C. B. Morrey, Jr., October 21, 1968

1. **Introduction.** The aims of this work were

(1) to establish a canonical theory of the nonparametric multiple integral problems with variable boundaries which is simpler than the Carathéodory theory [1] and contains fewer variables;

(2) to choose the canonical variables so that by specializing to simple integrals the theory reduces to the equations which are known from Carathéodory's textbook [2], and

(3) to suppose from the beginning that the problem is of the Lagrange type, i.e. that there is a set of differential equations which the extremals must satisfy.

The results obtained are stated in the following sections.

2. **The problem.** Our problem is to minimize the integral

$$(2.1) \quad \int \cdots \int F(t^\alpha, x^i, x^{i\alpha}) \prod_{\kappa=1}^{m} dt^\kappa \quad (i = 1, 2, \cdots, n, \ \alpha = 1, 2, \cdots, m)$$

taking into account the side conditions

$$(2.2) \qquad G_{\rho'}(t^\alpha, x^i, x^{i\alpha}) - \Gamma_{\rho'} = 0 \quad (\rho' = 1, 2, \cdots, p, \quad p < nm).$$

So, we number

(1) by greek indexes the independent variables,
(2) by latin indexes the dependent variables, and
(3) by greek indexes with accents the side conditions.

To speak more exactly, (2.1), (2.2) is a family of variational problems with the parameters $\Gamma_{\rho'}$. Suppose that originally the side conditions

$$(2.3) \qquad G_{\rho'}(t^\alpha, x^i, x^{i\alpha}) = 0$$

are given. Then, it turns out to be advantageous to replace them from the beginning by the family of side conditions (2.2) thus introducing the parameters $\Gamma_{\rho'}$ which later on are essential in the theory.

Throughout, we suppose that

$$(2.4) \qquad F > 0.$$

[1] Research work sponsored by the Deutsche Forschungsgemeinschaft.