A VARIATIONAL PROBLEM WITH INEQUALITIES AS BOUNDARY CONDITIONS

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Variational problems, for the Dirichlet integral and other functionals, with inequalities as boundary conditions have been discussed by G. Fichera [3], J. L. Lions and G. Stampacchia [8], H. Lewy [6], H. Brezis et G. Stampacchia [1], H. Lewy and G. Stampacchia [7]. The question what happens when an object is pushed against a soap film leads to such a problem for the area integrand. While it can readily be demonstrated that a suitably chosen minimizing sequence tends to a limit surface, this surface merely is a weak solution of the problem which in reality possesses farther-reaching and characteristic regularity properties. It may be of interest to note (and can easily be verified by experiments with needles) that an obstruction consisting of one point, or even of a compact set of points of vanishing linear measure, is not capable of lifting the soap film. This follows from the general maximum principle in [2], [10]; see also [11], [12]. In H. Lewy's paper [6] the special case is considered where the obstructing object consists of a plane convex curve, and it is shown that the solution surface (minimizing Dirichlet's integral) actually is continuous everywhere and attaches itself to the obstruction along precisely one interval. The purpose of the present note is to report a similar result for the problem of minimizing the area integral over a convex domain which is symmetric with respect to the plane of the obstructing convex curve. Details of the proof,[2] as well as applications of the method to more general cases, will appear elsewhere.

Let P be a bounded (open) convex domain in the (x, y)-plane, symmetric with respect to the x-axis and on this axis containing the closed segment $\sigma_0=\{x, y;\ |x|\leqq a,\ y=0\}$. Let P_0 be the set $P\backslash\sigma_0$ and $\overline{P}_0$ its closure; of course $\overline{P}_0=\overline{P}$. The boundary ∂P of P intersects the x-axis in the endpoints of an interval $\sigma=\{x, y;\ c_1\leqq x\leqq c_2,\ y=0\}$, where $c_1<-a$ and $c_2>a$. Denote by P^+ and P^- the intersections of P with the half planes $y>0$ and $y<0$, respectively. On σ_0 a continuous

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