ON SMOOTHNESS OF GENTLE PERTURBATIONS[1]

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1. **Introduction.** The notion of gentle perturbations was first introduced by K. O. Friedrichs in connection with some mathematical questions of the quantum theory of fields [1], while the theory of smooth perturbations was introduced by T. Kato in the study of wave operator in scattering theory [2]. It is naturally interesting to find out the relationship between these two concepts. In a recent paper [3, §5] the author was able to prove that every gentle operator of second kind (the class of integral operators with L^1 kernels) relative to some given operator T is a T-doubly smooth operator (see Definition 1.1 below). However, the question whether a T-gentle operator of first kind (the class of operators which can be represented as integral operators with Hölder continuous kernels in a representation in which T is diagonalized) is a T-doubly smooth operator remains unsettled. The purpose of this note is to announce a partial answer to this question. More specifically, we are able to give a simple direct proof that the class of selfadjoint gentle operators of finite rank and the class of nonnegative selfadjoint gentle operators are doubly smooth. Finally we apply our results to obtain some stability theorem on small perturbations.

Throughout this note let H, H' be separable Hilbert spaces and let T be a selfadjoint operator in H with its spectral family $\{E_T(\lambda)\}$. It is known (see [2, Theorem 5.1]) that a closed, densely defined operator A from H to H' with domain $D(A)\supset D(T)$ is T-smooth if $\|A\|_T < \infty$, where

$$\|A\|_T^2 = \sup\|AE_T(I)u\|^2/|I|\,\|u\|^2,$$

the suprema are taken over all $0\neq u\in H$ and all semiclosed finite interval $I=(a,b]$ with $|I|$ = the length of I and $E_T(I)=E_T(b)-E_T(a)$.

DEFINITION 1.1. Two closed, densely defined operators A, B from H to H' are said to form a T-doubly smooth pair, if they satisfy the following conditions.

(i) Both A and B are T-smooth, and

(ii) There exists a constant $N<\infty$ such that

$$\|A(T-z)^{-1}B^*u'\| \leq N\|u'\|, \qquad \operatorname{Im} z \neq 0,\ u' \in D(B^*),$$

where B^* denotes the adjoint operator of B.

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