DIFFERENTIABLE CLASSIFICATION OF SOME TOPOLOGICALLY LINEAR ACTIONS

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Communicated by William Browder, August 19, 1968

Let G act smoothly on S^n with fixed point set diffeomorphic to S^k $0 \leqq k \leqq n-3$, and with all other orbits of the same type and of dimension r. Connell, Montgomery and Yang have shown that the action of G is topologically equivalent to a linear action of S^n if $n-r \geqq 5$. The problem then arises as to what can be said about the differentiable classification of these topologically linear actions. In some special cases, the answers are known. For example, it was shown by Hirsch, Smale, and Livesay and Waldhausen that every smooth action of Z_2 on S^3 was differentiably equivalent to a linear one. Also, for high dimensional cases, the unknottedness of the codimensional 2 fixed point sphere of a smooth circle group action was shown to be true by Wu-Yi Hsiang. However, we prove that there is a curious involution on S^{23} with exactly two fixed points which is differentiably inequivalent to the linear action. Also we prove that for $i+1 \equiv 0 \pmod 4$, and if the codimension is bigger than 2 and lower than metastable range, there are always semifree circle group actions on the standard n-sphere S^n with the fixed point sets to be knotted i-spheres. The following are some of our main classification theorems.[2]

THEOREM 1. *Up to orientation preserving diffeomorphisms, the set of equivalence classes of semifree actions of S^1 on S^{2n+2} with exactly two fixed points forms a group and is the same as the subgroup of weak isotopy classes of diffeomorphisms of complex projective n-space CP^n which preserve the generator of $H^2(CP^n; Z)$.*

Similar classification theorems hold for semifree actions of cyclic P-group (P, an odd prime). However, for the case of smooth involutions with exactly two fixed points it is more complicated. Let f be a diffeomorphism of the real projective space RP^n to itself, which is required to be orientation preserving if n is odd, and let $Z_2 \to S^n \to RP^n$ be the universal covering over RP^n. By our assumption on f,

[1] Most of the results presented here appeared in the author's doctoral dissertation written under the direction of Professor Frank Raymond and submitted to the University of Michigan in partial fulfillment of the requirements for the Ph.D. degree.

[2] W. Browder has obtained similar classification theorems independently.