EUCLIDEAN n-PLANES IN PSEUDO-EUCLIDEAN SPACES AND DIFFERENTIAL GEOMETRY OF CARTAN DOMAINS

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1. **Introduction.** The Cartan domains, which we shall define in §3, include among them the four general types of (irreducible) bounded symmetric domains, first studied by E. Cartan [2], [3]. An (essentially unique) invariant Riemannian metric—the Bergman metric—exists on each of these bounded symmetric domains, and the resulting differential geometry has been studied by Siegel [7], Hua [4], [5], Look [6] and others.

In this note we describe how the differential geometry of Cartan domains can be studied neatly and effectively through a study of the Euclidean n-planes in a pseudo-Euclidean $(n+m)$-space of index m. Our results include a geometric interpretation of the Bergman metric, the theorem that domains of the second and third types are totally geodesic submanifolds of a domain of the first type, and ranges of value of the sectional curvature. Only a brief description of the method and results will be given here. The reader will find in this and three other notes [8], [9], [10] the essence of the differential geometry of the eight nonspecial types of irreducible Hermitian symmetric spaces (see [1]).

2. **Euclidean n-planes in a pseudo-Euclidean space.** Let F be the field R of real numbers, the field C of complex numbers, or the field H of real quaternions. Let $\{1, i, j, k\}$ be the usual basis of F over R. If $\xi=a_0+a_1i+a_2j+a_3k$, then

$$\bar{\xi} = a_0 - a_1i - a_2j - a_3k, \qquad \xi^\tau = a_0 + a_1i + a_2j - a_3k$$

are two conjugates of ξ. If A is an $n\times m$ matrix with elements in F, we denote by A^*, A^τ the two respective conjugate transposes of A. For a square matrix A, if $A^*=A$, $A^\tau=A$, or $A^\tau=-A$, we say, respectively, that A is Hermitian, τ-symmetric, or τ-skew-symmetric. Clearly, for $F=R$ or C, τ-symmetry and τ-skew-symmetry are the ordinary symmetry and ordinary skew-symmetry.

By definition, a pseudo-Euclidean space $F^{n+m}_{(m)}$ (of index m) is an $(n+m)$-dimensional left vector space over F provided with a (Hermitian) inner product $\langle\ ,\ \rangle$ such that there exist n-planes (i.e. n-dimensional vector subspaces), but not $(n+1)$-planes, on which the induced