PIECEWISE LINEAR GROUPS AND TRANSFORMATION GROUPS

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The object of this note is to state certain theorems which show that the notions of topological group and transformation group become unreasonably restricted when transposed to the piecewise linear category. Details will appear elsewhere.

Let us understand a *piecewise linear* (*PL*) *group* to be a topological group G together with a piecewise linear structure on G (i.e., triangulation of G as a locally finite simplicial complex), in terms of which the group multiplication $G\times G\rightarrow G$ and inversion $G\rightarrow G$ are given by piecewise linear functions.

THEOREM A. *The only connected PL groups are the abelian Lie groups*, $(S^1)^m\times R^n$, *and in general the only PL groups are extensions of these by discrete groups.*

This is an immediate consequence of

LEMMA. *Every PL group is locally PL isomorphic to the Euclidean group of the same dimension.*

The lemma is proved by choosing local PL coordinates in R^n for G, subdividing so that the multiplication map is linear on simplexes, and then examining the map in detail on an individual simplex.

THEOREM B. *Two PL groups are PL isomorphic if and only if they are topologically isomorphic, and any topological isomorphism between them is automatically a PL isomorphism.*

This also follows easily from the above lemma.

Now let G be a PL group, acting as a topological transformation group on the PL manifold M, via the map

$$F: G\times M\rightarrow M.$$

If F is a PL map, we say that G is a *PL transformation group* acting on M.

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