ON THE STABLE DIFFEOMORPHISM OF HOMOTOPY SPHERES IN THE STABLE RANGE, $n<2p$

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1. Introduction and statement of results. Let Θ_n^{p+1} denote the subgroup of the Kervaire-Milnor group θ_n of those homotopy n-spheres imbedding with trivial normal bundle in euclidean $(n+p+1)$-space $(n<2p)$. It is known that every homotopy n-sphere Σ^n imbeds in $(n+p+1)$-space with normal bundle independent of the imbedding provided, $n<2p$, [8]. Let $\Omega_{n,p}$ denote the quotient group θ_n/Θ_n^{p+1}.

It has been proved both by the author and R. DeSapio [3] that the order of $\Omega_{n,p}$, after identifying elements with their inverses, is just the number of diffeomorphically distinct products $\Sigma^n \times S^p$. It is shown in [3] that the *stable range* $n<2p$ is not necessary for the theorem. However, *it is crucial for all our own work on* $\Omega_{n,p}$. Indeed, it is in the stable range that the calculation of $\Omega_{n,p}$ is reducible to an effectively computable homotopy question. Further results on properties of $\Omega_{n,p}$, and in particular its relation to the determination of the number of smooth structures on $S^n \times S^p$, can be found in the very interesting work of DeSapio [3], [4] and [5].

From results of [8] it is immediate that $\Omega_{n,p}=0$ for $p \geqq n-3$ or $n \leqq 15$, $n<2p$ and $\Omega_{16,12}=Z_2$; the following theorems are extensions of these results for the stable range $n<2p$.

THEOREM 1.1. (i) $\Omega_{n,p}=0$ *if* $p \geqq n-7$ *and* $n \not\equiv 0, 1 \pmod 8$.
(ii) $\Omega_{n,n-4}=Z_2$ *for* $n=16, 32$.
(iii) $\Omega_{17,10}=Z_2$; $\Omega_{n,p}=0$ *if* $p \geqq n-6$ *and* $n \equiv 1 \pmod 8$.

Parts (ii) and (iii) show that (i) is best possible. However, we can also show

THEOREM 1.2. $\Omega_{n,n-13}=0$ *if* $n \equiv 4, 5 \pmod 8$.

Therefore, (i) of Theorem 1.1 is by no means the final answer. The table below gives our results for $n \leqq 20$.

Letting $\phi_n^{p+1}: \theta_n \to \pi_{n-1}(\mathrm{SO}(p+1))$ be the characteristic homomorphism of [8] we have

THEOREM 1.3. $\Omega_{n,p} = \operatorname{im} \phi_*^{p+1}$.

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