STURM COMPARISON THEOREMS FOR ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

BY J. B. DIAZ AND JOYCE R. McLAUGHLIN

Communicated September 30, 1968

1. **Introduction.** Comparison theorems for differential equations have attracted the attention of mathematicians for a long time. The first results of this nature, concerning linear, ordinary, second order, selfadjoint, differential equations $(ru')'+pu=0$, go back to Sturm [1]. Further developments along this line are due to M. Picone [2], and M. Bôcher [3], [4]; an account of this theory is to be found in the book of E. L. Ince [5]. The adjective "Sturm," in the title of this research announcement, is meant to refer, in a general way, to results similar to Sturm's, not only for single ordinary differential equations, but also for single partial differential equations and for systems of ordinary or partial differential equations.

Most of the "Sturm comparison theorems" announced here can be described roughly as follows: the hypothesis is that a certain nontrivial function is a solution of a given equation, and is zero on the boundary of a bounded, open, connected set, while the conclusion asserts that a solution of a related equation has a zero in the open set under consideration. In other Sturm comparison theorems announced here, the conclusion remains the same, while the hypothesis is now that a certain nontrivial function satisfies an integral inequality, and is zero on the boundary of a bounded, open, connected set. This integral inequality hypothesis is suggested by the work of W. Leighton [6]. The proof of both kinds of Sturm's theorems depends, in an essential manner, on identities similar to that of the Picone identity in the classical Sturm theory for ordinary (M. Picone [2]) and partial differential equations (M. Picone [7]).

2. **Ordinary differential equations.** In seeking to prove a "Sturm comparison theorem, for a single equation or a system of equations, under the weakest possible hypotheses," the following theorem was obtained.

THEOREM 1. *Suppose*

(1) *z and u^q, for $q=1, \cdots, m$, are differentiable vector valued functions with real components on $-\infty < a \leqq x \leqq b < +\infty$, $a \neq b$;*

(2) *r, $\bar{r}$, p, $\bar{p}$ are symmetric, m by m matrices with real valued components defined on $a \leqq x \leqq b$; $\bar{r}$ is positive definite for $a \leqq x \leqq b$; z and u^q*