BAXTER ALGEBRAS AND COMBINATORIAL IDENTITIES. II

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1. **Introduction.** The problem of obtaining an analog of the Baxter-Bohnenblust-Spitzer formula for cyclic permutations was suggested to me by Mark Kac. Its solution is presented here; the method leads in fact to similar identities for any group of permutations; the main result is identity (7) of §5. The tools of the proof are the result of I, which reduces computations with Baxter operators to computations with symmetric functions, and Möbius inversion on the lattice of periods of a group action (definition below).

To be sure, the present results are more of combinatorial than of probabilistic interest; further applications, with special regards to the Baxter algebras arising in probability, will be given in the third part.

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2. **Notation.** A partition π of a set S is a family of disjoint nonempty subsets of S, called *blocks*, whose union is S. Partitions are ordered by refinement: $\pi \leqq \sigma$ if every block of π is contained in one block of σ. The letter O denotes the partition where each block has one element. For the statement of the Möbius Inversion Formula we refer to the author's paper; little beyond the statement is needed. Bracketed statements and formulas denote partial results. Some details are omitted, but enough are given so that the full proof can be reconstructed.

3. **Generating functions.** A *function* is a function from a finite set of integers to the positive integers N. We associate to every function $f: S \to N$ a formal power series in the variables x_j^i, $i \in S$, $1 \leqq j < \infty$, as follows.

Let

$$M(f) = \prod_{i \in S} x_{f(i)}^{i};$$

if A is a set of functions, then let

$$M(A) = \sum_{f \in A} M(f).$$

Call $M(A)$ the *generating function* of the set A.