RESEARCH ANNOUNCEMENTS

BAXTER ALGEBRAS AND COMBINATORIAL IDENTITIES. I

BY GIAN-CARLO ROTA

Communicated August 15, 1968

1. **Introduction.** The spectacular results in the fluctuation theory of sums of independent random variables, obtained in the last 15 years by Andersen, Baxter, Bohnenblust, Foata, Kemperman, Spitzer, Takacs and others, have gradually led to the realization that the nature of the problem, as well as that of the methods of solution, is algebraic and combinatorial. After Baxter showed that the crux of the problem lay in simplifying a certain operator identity, several algebraic proofs (Atkinson, Kingman, Wendel) followed. It is the present purpose to carry this algebraization to the limit: the result we present amounts to a solution of the word problem for Baxter algebras. The solution is not presented as an algorithm, but by showing that every identity in a Baxter algebra is effectively equivalent to an identity of symmetric functions independent of the number of variables. Remarkably, the identities used so far in the combinatorics of fluctuation theory "translate" by the present method into classical symmetric function identities of easy verification. The present method is nevertheless also useful for guessing and proving new combinatorial identities: by way of example, it will be shown in the second part of this note how it leads to a generalization of the Bohnenblust-Spitzer formula for the action of arbitrary groups of permutations. A parallel theory of inequalities will be presented elsewhere.

2. **Definitions.** Let A be a commutative ring. A *Baxter operator* on A is a linear function P mapping all of A into itself and satisfying the identity

$$P(fPg) + P(gPf) = PfPg - P(fg). \tag{1}$$

The pair (A, P) will be called a *Baxter algebra*.

Let O be the category whose objects (A, T) are rings A together with operators $T: A \to A$ s.t. $T(f+g) = Tf + Tg$, and whose maps