DUALITY AND ASYMPTOTIC SOLVABILITY OVER CONES[1]

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Introduction. A duality theory for linear programming over closed convex cones is developed by using the solvability theorem of [1]. A complete classification of all duality states is given, which includes Duffin's duality theorem (1 of [3]), together with asymptotic refinements. For convenience, our results are stated in R^n, but extensions to more general spaces considered in [1] are possible. Complete proofs of all results will be given elsewhere. The main ingredients of the duality theory over closed convex cones are essentially due to Duffin as set forth in his fundamental paper [3]; see also Kretschmer [5].

0. **Notations and preliminaries.** C—a closed convex cone in R^n,

$$C^* = \{y \in R^n : x \in C \Rightarrow (y, x) \geqq 0\}.$$

For the system

$$Ax = b \qquad x \in C \tag{1}$$

with given $A \in R^{m \times n}$, $b \in R^m$ let

$$F(1) = \{x \in C : Ax = b\},$$

$$AF(1) = \{\{x_k : k = 1, 2, \cdots\} : x_k \in C, \lim_k Ax_k = b\}.$$

$AF(1)$ is a set of sequences. If $F(1) \neq \emptyset$ then it can be imbedded naturally in $AF(1)$.

DEFINITION. (1) is
CONS (consistent) if $F(1) \neq \emptyset$,
INC (inconsistent) if $F(1) = \emptyset$,
AC (asymptot. consist.) if $AF(1) \neq \emptyset$,
SINC (strongly inconsist.) if $AF(1) = \emptyset$.
We use the following

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