ON PROPERTIES OF SELF RECIPROCAL FUNCTIONS

BY V. V. L. N. RAO

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Following is the notation of Hardy and Titchmarsh [1]. We denote a function as $R\mu$ if it is self reciprocal for Hankel transforms of order μ, so that it is given by the formula

$$f(x) = \int_0^\infty J_\mu(xy) f(y) \sqrt{xy}\, dy, \tag{1.1}$$

where $J_\mu(x)$ is a Bessel function of order μ. For $\mu = \frac{1}{2}$ and $-\frac{1}{2}$, $f(x)$ is denoted as R_s and R_c respectively.

Brij Mohan [2] has shown that if $f(x)$ is $R\mu$, and

$$P(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} 2^s \Gamma\left(\frac{1}{4} + \frac{\mu}{2} + \frac{s}{2}\right)\Gamma\left(\frac{1}{4} + \frac{\nu}{2} + \frac{s}{2}\right)\theta(s) x^{-s} ds,$$

where

$$\theta(s) = \theta(1 - s) \quad \text{and} \quad 0 < c < 1, \tag{1.2}$$

then $P(x)$ is a Kernel transforming R_μ (R_ν) into R_ν (R_μ). As an example of (1.2) Brij Mohan has shown that the function

$$x^{\nu+1/2} e^{-x} \tag{1.3}$$

is a Kernel transforming R_ν into $R_{\nu+1}$. In particular, putting $\nu = \frac{1}{2}$, we find that the Kernel

$$x e^{-x}, \tag{1.4}$$

transforms R_s into $R_{3/2}$. Again, I have shown in a previous paper [3] that the Kernel

$$\sqrt{x} e^{-x/2}, \tag{1.5}$$

transforms R_1 into R_2. From (1.4) and (1.5) we find that "A Kernel transforming R_1 into R_2 will have its square transforming R_s into $R_{3/2}$." Again Sneddon [4] has shown that

$$\int_0^\infty e^{-x} x^m \mathrm{Ln}(x)\, dn = (-1)^m m! \int_0^\infty \frac{d^{n-m}}{dn^{n-m}} (x^n e^{-x})\, dx,$$

$\mathrm{Ln}(x)$ being Laguerre polynomial of order n. Putting $m = n$, we obtain that