A MAXIMUM PRINCIPLE FOR OPTIMAL CONTROL PROBLEMS WITH FUNCTIONAL DIFFERENTIAL SYSTEMS[1]

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In this note we present a maximum principle in integral form for optimal control problems with delay-differential system equations which also contain delays in the control. Recent related results for particular cases of the systems discussed below may be found in [1], [5], and [6]. Vector matrix notation will be used and we shall not distinguish between a vector and its transpose.

Let α_0 and t_0 be fixed in R^1 with $-\infty<\alpha_0<t_0$, $I=[\alpha_0, a)$ be a bounded interval containing $[\alpha_0, t_0]$, and put $I'=(t_0, a)$. For x continuous on I and t in I', the notation $F(x(\cdot), t)$ will mean that F is a functional in x, depending on any or all of the values $x(\tau)$, $\alpha_0\leqq\tau\leqq t$. $\overline{\Phi}$ will denote the class of absolutely continuous $n-1$ vector functions on $[\alpha_0, t_0]$. Let Ω be a given convex subset of the class of all bounded Borel measurable functions u defined on I into R^r, and $\mathfrak{J}$ be a given C^1 manifold in R^{2n-1}. The problem considered is that of minimizing

$$J[\bar{\phi}, u, \bar{x}, t_1] = \int_{t_0}^{t_1} f^0(\bar{x}(\cdot), u(\cdot), t)dt$$

over $\overline{\Phi}\times\Omega\times C(I, R^{n-1})\times I'$ subject to

$$\text{(i)}\quad \dot{\bar{x}}(t) = \bar{f}(\bar{x}(\cdot), u(\cdot), t) \qquad \text{a.e. on } [t_0, t_1],$$
$$\bar{x}(t) = \bar{\phi}(t) \qquad \text{on } [\alpha_0, t_0],$$
$$\text{(ii)}\quad (\bar{x}(t_0), \bar{x}(t_1), t_1) \in \mathfrak{J}.$$

We assume that $f=(f^0, \bar{f})=(f^0, f^1, \cdots, f^{n-1})$ is an n-vector functional of the form

$$f^i(\bar{x}(\cdot), u(\cdot), t) = h^i(\bar{x}(\cdot), t) + \int_{\alpha_0}^{t} u(s)d_s\eta(t, s)g^i(\bar{x}(s), t) \tag{1}$$
$$\text{for } i = 0, 1, \cdots, n-1,$$

where the integral is a Lebesgue-Stieltjes integral. Each $h^i(\bar{x}(\cdot), t)$ is

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