EXTENDING COHERENT ANALYTIC SHEAVES THROUGH SUBVARIETIES

BY YUM-TONG SIU[1]

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We announce the following

THEOREM I. *Suppose V is a subvariety of dimension $\leqq n$ in a (not necessarily reduced) complex space X and $\mathfrak{F}$ is a coherent analytic sheaf on $X-V$ with* codh $\mathfrak{F} \geqq n+3$. *Let $\theta: X-V \to X$ be the inclusion map. Then $\theta_0(\mathfrak{F})$ is a coherent analytic sheaf on X extending $\mathfrak{F}$ (where $\theta_q(\mathfrak{F})$ is the qth direct image of $\mathfrak{F}$ under θ).*

The case $n=0$ was proved in [7]. The case where X is a manifold of dimension $n+3$ was proved in [5].

We give here only a very brief outline of the proof together with some related results and application. Details will appear elsewhere.

Suppose $\mathfrak{F}$ is an analytic sheaf on a complex space X and n is a nonnegative integer. We denote by $\mathfrak{F}^{[n]}$ the analytic sheaf on X defined by the following presheaf: if $U \subset W$ are open subsets of X, then $\mathfrak{F}^{[n]}(U)=$ the direct limit of $\{\Gamma(U-A, \mathfrak{F}) \mid A \in \mathfrak{A}\}$, where $\mathfrak{A}$ is the set of all subvarieties of dimension $\leqq n$ in U directed by inclusion, and $\mathfrak{F}^{[n]}(W) \to \mathfrak{F}^{[n]}(U)$ is induced by restriction maps. There is a canonical sheaf-homomorphism from $\mathfrak{F}$ to $\mathfrak{F}^{[n]}$. We denote by $O_{[n]}\mathfrak{F}$ the analytic subsheaf of $\mathfrak{F}$ defined as follows: for $x \in X$, $s \in (O_{[n]}\mathfrak{F})_x$ if and only if there exist an open neighborhood U of x in X, a subvariety A in U of dimension $\leqq n$, and $t \in \Gamma(U, \mathfrak{F})$ such that $t_x = s$ and $t_y = 0$ for $y \in U-A$.

PROPOSITION 1 [6]. *Suppose $\mathfrak{F}$ is a coherent analytic sheaf on a complex space X and n is a nonnegative integer.*

(a) *If $O_{[n+1]}\mathfrak{F}=0$, then $\mathfrak{F}^{[n]}$ is coherent and the subvariety where $\mathfrak{F}^{[n]}$ is not isomorphic to $\mathfrak{F}$ canonically is of dimension $\leqq n$.*

(b) *If $\mathfrak{F}$ is canonically isomorphic to $\mathfrak{F}^{[n]}$, then $O_{[n+1]}\mathfrak{F}=0$.*

The following can be proved from Proposition 1 and by induction on n.

PROPOSITION 2. *Suppose $\mathfrak{F}$ is a coherent analytic sheaf on a complex space X and n is a nonnegative integer such that $\mathfrak{F}$ is canonically iso-*

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