

FUNCTIONS OF FINITE λ -TYPE IN SEVERAL COMPLEX VARIABLES

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In [1] L. A. Rubel and B. A. Taylor present some results in the theory of meromorphic and entire functions of finite λ -type on the plane. By applying Fourier series techniques they are able to achieve a generalization of a classical theorem of Lindelöf concerning the existence of an entire function of restricted order and type having a prescribed set of zeros. In [2] Wilhelm Stoll develops a generalization of this theorem of Lindelöf for the case of functions of several variables using the techniques of value distribution theory. The purpose of this note is to report some results toward a comprehensive theory of functions of finite λ -type in several complex variables which are produced by combining the techniques of Rubel and Taylor with those of Stoll.

A continuous nondecreasing function from the positive real numbers into the positive real numbers is called a *growth* function. If λ is a growth function, then a meromorphic function f on \mathbf{C}^k is said to be of *finite λ -type* if the upper limit at infinity of the quotient $T(r, s; f)/\lambda(Br)$ is finite for some positive constants s and B , where $T(r, s; f)$ is the (Kneser-Stoll) characteristic of f for $r \geq s \geq 0$. If, in particular, f is entire, then f is of finite λ -type if and only if there are positive constants A , B , and R such that $|f(\zeta)| \leq \exp(A\lambda(B|\zeta|))$ for all ζ in \mathbf{C}^k with $|\zeta| > R$.

The class of all meromorphic functions on \mathbf{C}^k which are of finite λ -type for a given λ is a field under the usual operations on functions. Moreover, this field is invariant under parallel translations of the variable in \mathbf{C}^k and is either the class of constant functions or contains all rational functions on \mathbf{C}^k . Similarly, the class of all entire functions on \mathbf{C}^k which are of finite λ -type for a given λ is an integral domain. These facts are immediate consequences of the properties of the characteristic. A natural question, then, is the following: For which growth functions λ is the field of meromorphic functions of finite

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