ON GENERATORS FOR VON NEUMANN ALGEBRAS[1]

BY WARREN WOGEN

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1. It has been conjectured that every von Neumann algebra on a separable Hilbert space has a single generator. The conjecture is true for type I algebras [3] and for hyperfinite algebras [7, Theorem 1].

T. Saitô [6] showed recently that for a certain class of von Neumann algebras, every algebra generated by two operators has a single generator. We show in §2 of this paper that every finitely generated algebra of the class has a single generator. In §3, we prove that every properly infinite von Neumann algebra on a separable Hilbert space is singly generated.

Throughout this paper, $\mathcal{H}$ will denote a separable complex Hilbert space. Operator always means bounded linear operator on a Hilbert space. $\mathcal{B}(\mathcal{H})$ is the set of bounded linear operators on $\mathcal{H}$. If $\mathcal{A}$ is a von Neumann algebra, then $\mathcal{A}'$ is the commutant of $\mathcal{A}$, and for $2\leqq n\leqq\aleph_0$, $M_n(\mathcal{A})$ is the algebra of $n\times n$ matrices with entries in $\mathcal{A}$ which act boundedly on $\sum_{i=1}^{n}\oplus\mathcal{H}$. $\mathcal{R}(A, B, \cdots)$ denotes the von Neumann algebra generated by the family $\{A, B, \cdots\}$ of operators.

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2. If $\mathcal{A}$ is a von Neumann algebra, let $(*)$ be the property that $\mathcal{A}$ is $*$-isomorphic to $M_2(\mathcal{A})$. We will prove the following

THEOREM 1. *Let $\mathcal{A}$ be a von Neumann algebra which satisfies $(*)$ and suppose that $\mathcal{A}$ is finitely generated. Then $\mathcal{A}$ has a single generator.*

The following lemmas are needed in the proof of the theorem. These lemmas are generalizations of lemmas proved by T. Saitô in [6].

LEMMA 1. *Suppose a von Neumann algebra $\mathcal{A}$ is generated by n operators $A_1, A_2, \cdots, A_n$, $n\geqq 2$. Then $M_2(\mathcal{A})$ is generated by the $n+1$ operators*

$$\begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} A_2 & 0 \\ 0 & 0 \end{pmatrix}, \cdots, \begin{pmatrix} A_n & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}.$$

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