IDEALS IN GROUP ALGEBRAS

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Throughout this note G denotes a locally compact abelian group and a Hausdorff space. The ideal structure of the group algebra $L_1(G)$ is still not fully known. For example, at a recent international symposium on functional analysis held at Sopot, Poland, the following questions were asked: (i) find maximal nonclosed ideals in $L_1(G)$ and (ii) what type of prime ideals are in $L_1(G)$? The following theorems answer these questions.

THEOREM 1. *Every maximal ideal of G is regular and, therefore, closed.*

In view of Theorem 1 we have the following

LEMMA 1. *If I is an ideal in $L_1(G)$ such that I is contained in exactly one maximal ideal, say M, then $\bar{I}=M$ ($\bar{I}$ is the closure of I).*

LEMMA 2. *If a prime ideal I of $L_1(G)$ is contained in a maximal ideal, then I is contained in only one maximal ideal.*

LEMMA 3. *If I is an ideal of $L_1(G)$ such that I is contained in no maximal ideal, then $\bar{I}=L_1(G)$.*

LEMMA 4. *Suppose I is a prime ideal of $L_1(G)$ such that I is contained in no maximal ideal and M is a maximal ideal in $L_1(G)$. If $J=I\cap M$, then $\bar{J}=M$ (this holds for every M).*

THEOREM 2. *If I is a prime ideal in $L_1(G)$, then I is maximal if and only if I is closed.*

Theorems 1 and 2 stated above answer questions raised at the Sopot symposium. In what follows $\hat{G}$ denotes the dual group of G.

THEOREM 3. *If $\hat{G}$ contains an infinite set, then $L_1(G)$ contains nonclosed prime ideals.*

(By the previous theorem each one is nonmaximal. The converse is true. See Corollary 4 below.)

THEOREM 4. *The following two statements are equivalent.*

(1) *Each prime ideal is contained in a unique maximal ideal.*

(2) *G is a discrete group.*

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