BANACH ALGEBRAS OF OPERATORS ON LOCALLY CONVEX SPACES

BY ROBERT T. MOORE[1]

Communicated by H. Helson, July 23, 1968

1. **Introduction.** This is the first of three announcements of results concerning operator theory on locally convex topological vector spaces [3], [4]. The method described here uses certain well-behaved "finite" operators on such a space to study the properties of general operators acting there. Under suitable completeness conditions, these well-behaved operators appear in certain geometrically-determined Banach algebras; this fact brings to bear the highly developed classical theory of Banach algebras in the study of general operators on general locally convex spaces. Some of the methods are formally identical to those introduced by Allan in his earlier spectral theory for locally convex algebras [1].

Two principal applications have motivated the early research reported here and other work now in progress. It would be useful to have an operator theory on locally convex spaces of test functions and distributions powerful enough to treat problems in partial differential and "integro-differential" equations systematically within the framework of functional analysis. The infinite-dimensional representation theory of Lie groups presents a related set of problems, when treated from a systematically "infinitesimal" point of view, on special locally convex spaces of "C^∞ vectors." The simplest sorts of applications of this type are illustrated in [4].

Full proofs of these and related results will appear in [5].

2. **Calibrations and normed algebras.** The notions and results discussed in this section depend upon the choice of a fixed "geometrical" structure on a Hausdorff locally convex topological vector space (lcs) $\mathfrak{X}$ over the ground field $F=\{R \text{ or } C\}$: a *calibration* Γ for $\mathfrak{X}$ is a collection of continuous seminorms p on $\mathfrak{X}$ which induces the topology of $\mathfrak{X}$. Then the collection of all

$$N(\epsilon, p) = \{u \in \mathfrak{X} \mid p(u) \leqq \epsilon\} \qquad \epsilon > 0, \quad p \in \Gamma$$

forms a neighborhood subbase at 0.

[1] Research supported in part by NSF GP 5585.