STRONGLY NEGLIGIBLE SETS IN FRÉCHET MANIFOLDS

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Let s denote the linear metric space which is the countable infinite product of lines. It is known [1] that s is homeomorphic to Hilbert space l_2 and, in light of [8] and [10], to all separable infinite-dimensional Fréchet spaces (and therefore, of course, to all such Banach spaces). We define a Fréchet *manifold* or *F-manifold* to be a separable metric space which admits an open cover by sets homeomorphic to open subsets of s. Banach manifolds, which may be similarly defined, have been studied by a number of authors. From the results cited above it follows that all separable metric Banach manifolds modeled on separable infinite-dimensional Banach spaces are, in fact, F-manifolds. Also, clearly, any open subset of an F-manifold is an F-manifold.

In this paper, we are concerned with homeomorphisms of F-manifolds onto dense subsets of themselves. The first result of the type we consider was due to Klee [11], who showed that for any compact set K in l_2, l_2 is homeomorphic to $l_2\backslash K$. Recently, there have been a number of results [2], [3], [4], [5], [7], [13], etc., showing that for various types of subsets K of certain linear metric spaces X, X is homeomorphic to $X\backslash K$. Bessaga [7] introduced the term "negligible" for such sets K. In some cases K was assumed compact, in others σ-compact (i.e. the countable union of compact sets) and in others K was assumed to be the countable union of closed sets of infinite deficiency (i.e. of infinite codimension). Indeed several different geometric methods [2], [3], [5], [7], [11] have been used to establish negligibility in various spaces. The results that σ-compact subsets of l_2 and of s are negligible were used in the proofs [1] and [5] that l_2 is homeomorphic to s. Questions of negligibility of subsets in Fréchet and Banach manifolds have also arisen. Where differentiable structures are assumed as for Banach spaces and manifolds and K is assumed closed, Bessaga [7], Corson, Eells and Kuiper [9], Kuiper and Burghelea [12], Moulis [13], Renz [15] and West have investigated conditions under which X and $X\backslash K$ are diffeomorphic,

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