MANIFOLDS HOMEOMORPHIC TO SPHERE BUNDLES OVER SPHERES

BY R. DE SAPIO[1]

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1. Statement of results. Let E be the total space of a k-sphere bundle over the n-sphere with characteristic class $\alpha \in \pi_{n-1}(SO_{k+1})$. We consider the problem of classifying, under the relation of orientation preserving diffeomorphism, all differential structures on E. It is assumed that E is simply connected, of dimension greater than five, and its characteristic class α may be pulled back to lie in $\pi_{n-1}(SO_k)$ (that is, the bundle has a cross-section). In [1] and [2] we gave a complete classification in the special case where $\alpha = 0$. The more general classification Theorems 1 and 2 below include this special case. The proofs of these theorems are sketched in §2 below; detailed proofs will appear elsewhere. J. Munkres [6] has announced a classification up to concordance of differential structures in the case where the bundle has at least two cross-sections. (It is well known that concordance and diffeomorphism are not equivalent, concordance of differential structures being strictly stronger than diffeomorphism.)

THEOREM 1. *Let E be the total space of a k-sphere bundle over the n-sphere whose characteristic class[2] α may be pulled back to lie in $\pi_{n-1}(SO_k)$. Suppose that $2 \leqq k < n-1$. Then, under the relation of orientation preserving diffeomorphism, the diffeomorphism classes of manifolds homeomorphic to E are in a one-to-one correspondence with the equivalence classes on the set $(\theta_n/\Phi_n^{k+1}) \times \theta_{n+k}$, where (A_*^n, U^{n+k}) and (B_*^n, V^{n+k}) are equivalent if and only if $A_*^n = \pm B_*^n$ and there exists $\beta \in \pi_k(SO_{n-1})$ such that $U^{n+k} - V^{n+k} = \tau'_{n,k}(A_*^n \otimes \beta) + \sigma_{n-1,k}(\alpha \otimes \beta)$.*

Theorem 1 is also true in the case where $k = n-1$ and n is odd. The classification in the case where $n-1 \leqq k \leqq n+2$ is essentially the same as the above and is given in Theorem 2 below. Now we establish the notation used in Theorem 1.

NOTATION. Manifolds and diffeomorphisms are of class C^∞. The group of homotopy n-spheres under the connected sum operation +

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[2] *Added in proof.* Assume here and in Proposition 2 that α is of order 2 in $\pi_{n-1}(SO_{k+1})$ in the case where $k < n-3$. This assumption is not made elsewhere.