PERIODIC ORBITS OF HYPERBOLIC DIFFEOMORPHISMS AND FLOWS[1]

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Artin and Mazur in [1] proved that a dense subset of the C^r-endomorphisms of a compact differentiable manifold satisfy an exponential growth condition on their isolated periodic points, and they defined a ζ-function which for these endomorphisms has a positive radius of convergence. In [2] and [3] K. Meyer gave a simple proof that hyperbolic diffeomorphisms and flows of Smale [4] which are C^2 have exponential growth. It is the purpose of this note to give an even simpler proof of Meyer's theorems in a C^1 setting. Since the hyperbolic diffeomorphisms and flows are not dense [5] these results are a long way from including the results of [1].

Let M be a compact differentiable manifold; let $f \in \mathrm{Diff}(M)$ be a C^1 diffeomorphism, and let $N_m(f)$ be the number of periodic points of f of period m.

THEOREM 1. *Let f satisfy Axiom* A *of* [4, I.6], *then there exist constants c and k such that $N_m(f) \leqq ck^m$.*

PROOF. f is expansive [4, I.8.7], i.e., $\exists \epsilon > 0$ such that given x, y distinct periodic points of $f \, \exists n \in Z$ such that $d(f^n(x), f^n(y)) \geqq \epsilon$. Since f is C^1 it is Lipshitz. Let its Lipshitz constant be k which we may choose >1. If x and y are both of period p we may choose n in $0 \leqq n < p$ and have $d(x, y) \geqq \epsilon / k^{p-1}$ by expansiveness. Thus there exists a constant c such that $N_p(f) \leqq c V(M)(2k^{p-1}/\epsilon)^{\dim M}$ where $V(M)$ is the volume of M.

Let $\Phi = \{\phi_t\}$ be a one parameter group acting on M, arising from a C^1 vector field X. Let $N_\tau(\Phi)$ be the number of closed orbits of Φ of period less than or equal to τ.

THEOREM 2. *Let Φ satisfy Axiom* A′ *of* [4, 5.1], *then there exist constants c and k such that $N_\tau(\Phi) \leqq ce^{k\tau}$.*

Since the closed orbits are uniformly bounded away from the singularities, which are finite in number, Ω_c the complement of the singularities in Ω, is compact. Every point z in Ω_c has a flow box

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