A FACTOR THEOREM FOR FRÉCHET MANIFOLDS

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Communicated by R. D. Anderson, August 9, 1968

1. Introduction. A Frechet manifold (or F-manifold) is a separable metric space M having an open cover of sets each homeomorphic to an open subset of the countable infinite product of open intervals, s. A Q-manifold is a separable metric space M having an open cover of sets each homeomorphic to an open subset of the Hilbert cube, I^{∞} . It is known that all separable metric Banach manifolds modeled on separable infinite-dimensional Banach spaces are F-manifolds. The following are the principle theorems of this paper.

THEOREM I. If M is any F-manifold, then $s \times M$ is homeomorphic to M.

THEOREM II. If M is any Q-manifold, then $I^{\infty} \times M$ is homeomorphic to M.

Since s is known, [1] or [3], to be homeomorphic to $s \times I^{\infty}$, from Theorem I we immediately have the following.

COROLLARY. If M is any F-manifold, then $I^{\infty} \times M$ is homeomorphic to M.

Almost identical proofs of Theorems I and II can be given. To emphasize the ideas of our proofs of Theorems I and II we shall outline instead a proof of the similar but notationally easier

THEOREM I'. If M is any F-manifold and J^0 is the open interval (-1, 1), then $J^0 \times M$ is homeomorphic to M.

2. Lemma 2.1 implies Theorem I'.

DEFINITION. Let r be a map, i.e. continuous function, of a space X into the closed unit interval [0, 1]. Let $J^0(0) = \{0\}$ and for $t \in (0, 1]$, let $J^0(t) = (-t, t)$. Then $J^0 \times^r X = \{(y, x) \in J^0 \times X : y \in J^0(r(x))\}$ is the variable product of J^0 by X (with respect to r).

LEMMA 2.1. Let U be an open subset of s, let $V \subset W \subset U$ where W is open and V is closed in U, and let $J^0 \times^{r_0} U$ be a variable product of J^0

¹ This paper is a brief resume of a paper *Factors of infinite-dimensional manifolds*, submitted by the authors to Trans. Amer. Math. Soc. The research was supported in part under NSF grants GP 6867 and GP 8637.