REGARDING STOPPING RULES FOR BROWNIAN MOTION AND RANDOM WALKS[1]

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At the Fifth Berkeley Symposium on Mathematical Statistics and Probability, A. Dvoretzky [4] presented a paper on "Certain Optimal Stopping Rules." In this paper he proved the existence of an optimal stopping rule for the following situation: let $X_1, X_2, X_3, \cdots$ be a sequence of independent, identically distributed, real-valued random variables with zero means and unit variances defined on the probability space $(\Omega_1, \mathfrak{F}_1, P_1)$. Then for (fixed) $\beta > \frac{1}{2}$, there exists a positive, integer-valued random variable n (called a stopping rule) defined on Ω_1, whose value for any sample point ω is a function only of the observable variables $X_1(\omega), X_2(\omega), \cdots, X_{n(\omega)}(\omega)$ and which realizes the maximum value of $E\{n^{-\beta} \sum_{i=1}^{n} X_i\}$ over all such random variables, i.e. stopping rules. In his proof, Dvoretzky was lead to conjecture concerning the asymptotic behavior of the stopping boundary which provided the definition of his optimal rule. Theorem 1 of this announcement provides an explicit description of the asymptotic behavior of this boundary. Theorem 2 gives an optimal stopping rule for an analogous situation for a continuous time process, namely Brownian motion.

The paper of Y. S. Chow and H. Robbins [2], which initiated work in this area of stopping rule problems, and the paper of H. Teicher and J. Wolfowitz [7] are also related to the results announced herein. Many of the ideas used to achieve these results were suggested by techniques exhibited in the paper by Teicher and Wolfowitz. While completing the writeup of the results announced herein, the work of L. A. Shepp [5] came to the attention of the author, in which Shepp obtained the same results as given by Theorem 1 and Theorem 2 for the case when $\beta = 1$.

Concise statements of these two theorems require the definition of some notation. The sequence $X_1, X_2, X_3, \cdots$ is as suggested above,

[1] Complete proofs of all statements made in this announcement can be found in the author's Ph.D. dissertation, *Stopping rules for Brownian motion and random walks*, University of California at Los Angeles, 1968, which was written under the direction of Professor C. J. Stone and was supported in part by the National Science Foundation through grant GP-5224. Some generalizations of the results presented in this announcement are also contained in the dissertation.