ON GAUSSIAN SUMS

BY TAKASHI ONO

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This note is an outline of some of the author's recent work on a generalization of Fourier transforms in adele spaces. Here we treat only the simplest case. The details and a generalization for an arbitrary ground A-field and a system of polynomials will be given elsewhere. For the unexplained notions, see [1], [2] and [3].

Let $f(X)$ be an absolutely irreducible polynomial in $Q[X] = Q[X_1, \cdots, X_n]$ such that the corresponding hypersurface $H = \{x \in \Omega^n; f(x) = 0\}$ is nonsingular, where Ω denotes a universal domain containing Q. Let V be the complement of H in Ω^n viewed as an algebraic variety in Ω^{n+1} in an obvious way. Hence the n-form $\omega = f^{-1}dx$, $dx = dx_1 \wedge \cdots \wedge dx_n$, is everywhere holomorphic and never zero on V. For each valuation v of Q, denote by Q_v the completion of Q at v. Denote by A, A^* the adele ring and the idele group of Q, respectively. For an idele $a \in A^*$, $|a|_A$ will denote the module of a. The adelization V_A of V is then given by $V_A = \{x \in A^n; f(x) \in A^*\}$. We denote by $\mathcal{S}(Q_v^n)$, $\mathcal{S}(A^n)$ the space of Schwartz functions on Q_v^n, A^n, respectively. For each v, the n-form ω on V induces a measure ω_v on V_{Q_v} and we know that there is a well-defined measure dV_A on V_A of the form $\prod_v \lambda_v^{-1}\omega_v$ with $\lambda_\infty = 1$ and $\lambda_p = 1 - p^{-1}$. We know that the function

$$(1) \qquad Z(f, \phi, s) = \int_{V_A} \phi(x) \, |f(x)|_A^s dV_A, \quad \phi \in \mathcal{S}(A^n),$$

represents a meromorphic function for Re $s > \frac{1}{2}$ having the single simple pole at $s = 1$ with the residue $\int_{A^n} \phi(x) dA^n$, where dA^n is the canonical measure on A^n (cf. [4]).

Let χ be a basic character of A which identifies the additive group A with its own dual and let χ_v be the similar character of the additive group Q_v induced by χ. For each $\xi \in A$ and $\phi \in \mathcal{S}(A^n)$, the function $\phi_\xi(x) = \phi(x)\chi(f(x)\xi)$ is again in $\mathcal{S}(A^n)$ and hence we have

$$(2) \qquad \operatorname*{Res}_{s=1} Z(f, \phi_\xi, s) = \int_{A^n} \phi(x)\chi(f(x)\xi) dA^n \overset{\text{def.}}{=\!=} \mathcal{G}_f\phi(\xi).$$

The transform $\phi \to \mathcal{G}_f\phi$ is a linear map of $\mathcal{S}(A^n)$ into the space of con-