AN ALGEBRA OF SINGULAR INTEGRAL OPERATORS WITH TWO SYMBOL HOMOMORPHISMS

BY H. O. CORDES[1]

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1. Let $R_+^{n+1} = \{(x, y) = (x_1, \cdots, x_n, y) : x_j, y \in R, y \geqq 0\}$ and let Δ_d, Δ_n denote the two unbounded positive self-adjoint operators of the Hilbert-space $\mathfrak{H} = \mathcal{L}^2(R_+^{n+1})$ generated by closing the Laplace operator in $C_0^\infty(R_+^{n+1})$ under Dirichlet and Neumann boundary conditions at $y=0$, respectively.

We propose to study the "convolution algebra" $\mathfrak{A}^\#$ generated by the generalized Riesz-Hilbert-operators

$$\begin{aligned} &\Lambda_d = (1 - \Delta_d)^{-1/2}, \quad \Lambda_n = (1 - \Delta_n)^{-1/2}, \quad S_d = - i\partial/\partial y \Lambda_d, \\ (1) \qquad &S_n = - i\partial/\partial y \Lambda_n, \quad S_{d,j} = - i\partial/\partial x_j \Lambda_d, \quad S_{n,j} = - i\partial/\partial x_j \Lambda_n, \\ &\qquad\qquad j = 1, \cdots, n \end{aligned}$$

and later on also will adjoin certain multiplications by continuous functions, to obtain an algebra $\mathfrak{A}$ of singular integral operators on the half-space R_+^{n+1}.

Both C^*-algebras $\mathfrak{A}^\#$ and $\mathfrak{A}$ have noncompact commutators, but each is commutative modulo a certain larger ideal ($\mathfrak{E}^\#$ and $\mathfrak{E}$, respectively). We therefore obtain a first symbol function σ_A for $A \in \mathfrak{A}^\#$ (or $\mathfrak{A}$) which is a continuous complex-valued function over the maximal ideal space of $\mathfrak{A}^\#/\mathfrak{E}^\#$ (or $\mathfrak{A}/\mathfrak{E}$). If σ_A does not vanish, we can invert the operator mod $\mathfrak{E}^\#$ (or $\mathfrak{E}$), or reduce the singular integral equation $A_n u = f$ to an equation $(1+E)u = g$ with $E \in \mathfrak{E}^\#$ (or $\mathfrak{E}$).

Now, we find that the ideals $\mathfrak{E}^\#$ and $\mathfrak{E}$ are isomorphic to topological tensor products of the form $\mathfrak{C}(\mathfrak{h}) \hat{\otimes} \mathfrak{S}^\#$, $\mathfrak{E} = \mathfrak{C}(\mathfrak{h}) \hat{\otimes} \mathfrak{S}$, with respect to a suitable direct decomposition

$$\mathfrak{H} = \mathfrak{h} \otimes \mathfrak{k}, \qquad \mathfrak{h} = \mathcal{L}^2(R^+), \qquad \mathfrak{k} = \mathcal{L}^2(R^n),$$

where $\mathfrak{C}(\mathfrak{h})$ denotes the compact ideal of $\mathfrak{h}$, while $\mathfrak{S}^\#$ and $\mathfrak{S}$ are certain algebras of singular integral operators over the boundary R^{n+1}.

Therefore to each operator $E \in \mathfrak{E}^\#$ (or $\mathfrak{E}$) there can be associated an operator valued symbol $\tau_E(m) \in \mathfrak{C}(\mathfrak{h})$ such that $1+E$ is Fredholm if and only if $1+\tau_E(m)$ is regular for all m. The construction of a Fredholm inverse for $A \in \mathfrak{A}$ will therefore depend on two symbols: first we invert the operator modulo $\mathfrak{E}$, if the complex-valued symbol

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