A SIMPLE MODEL OF THE DERIVATION OF FLUID MECHANICS FROM THE BOLTZMANN EQUATION[1]

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1. **Introduction.** Numerous problems arising from nature can be described by a (possibly nonlinear) parabolic equation of the form

$$\text{(1a)} \qquad \partial p/\partial t = Q[p] \qquad (t > 0),$$

$$\text{(1b)} \qquad \lim_{t \downarrow 0} p = f.$$

Q is independent of $t > 0$ and maps a function f belonging to a certain manifold M into a tangent vector $Q[f]$ based at f, so that the flow defined by (1) is a flow *on* M.

Now it may happen that the solutions of (1) come rapidly close to a special submanifold H in which a striking simplification takes place: namely, the amount of information needed to distinguish points of H is suddenly much smaller than in the ambient manifold M, and owing to this simplification of the function space, the flow defined by (1) can be described much more simply. This phenomenon is familiar to students of statistical mechanics, notably in the passage pass from Boltzmann's equation to hydrodynamics via the Chapman-Enskog expansion; see, for instance, Ford-Uhlenbeck [2].

I will speak about 3 examples of this state of affairs. The first is due to Carleman [1, p. 106]. He attaches no particular significance to it, but it is cute and illustrates some of the ideas involved. The second example is the actual Chapman-Enskog-Hilbert development for the Boltzmann equation, or at least a conjecture as to how it should go; see §3 below. Boltzmann's equation is too complicated to prove very much about, but in the simplified model of §4 it is possible to compute everything and to see explicitly all the phenomena that the Boltzmann equation is supposed to exhibit.[2] Grad [3] discussed the Chapman-Enskog-Hilbert development for the linear approximation to the Boltzmann equation. This is still very complicated, so naturally the results are less satisfactory.

2. **Carleman's example.** Carleman, in his study of the Boltzmann equation [1], used the problem

[1] An address delivered before the New York meeting of the Society by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, April 12, 1968; received by the editors September 20, 1968.

[2] This material is taken from McKean [6].